LEARNING BETWEEN THE LINES:
A SYNCRETISTIC EXPERIMENT IN MATHEMATICS AND VISUAL ARTS EDUCATION

DANIEL H. JARVIS

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NIPISSING UNIVERSITY
FACULTY OF EDUCATION
NORTH BAY, ONTARIO

ABSTRACT

This Action Research study examines students' perceptions of integrated learning, and documents their performance on three such assignments. It also seeks to define some of the qualities germane to the integrated learning process, including key criteria for developing integrated curriculum. Utilizing both a qualitative and quantitative research design, information was gathered during a secondary school semester, through a variety of research instruments. These include three integrated mathematics and visual arts assignments, video-taped classroom presentations, student questionnaires, student interviews, a final student survey, and an Open House for parents/guardians. In the role of participant-observer, I was also able to record ongoing personal reflections in the form of field notes.

The rationale for the research seeks to furnish a multi-faceted justification of an integrated approach to education. The literature review examines the two key issues of curricular organization and integrated curriculum implementation. Related research studies regarding integration are compared to, and contrasted with, this present study.

The findings of this research indicate that students, having completed the three assignments, possessed the following perceptions of integrated learning: (i) that the application of mathematics, as experienced in the creation of SMARTWORKS (Secondary Math and Art Works), served to considerably reinforce the new mathematics learning; (ii) that the communication of mathematics, as experienced in both the written descriptions and the oral presentations, served to somewhat reinforce the new mathematics learning; (iii) that motivation was enhanced due to the unique and varied aspects of the assignments, and (iv) that this type of integrated approach to mathematics education should be used regularly. The findings also

indicate that these perceptions were shared by students of different gender, ages, and mathematical ability.

Within the framework of the integrated assignments, students favoured the elements of SMARTWORK creation, cooperative learning, exhibition of student work, and the Open House; while indicating less interest in Internet and home research, *Geometer's Sketchpad* connections, and oral presentations.

Assessment of student assignments revealed overall weaknesses in the Achievement Chart categories of *Mathematical Communication* and *Thinking/Inquiry/Problem Solving* while indicating overall strengths in the categories of *Knowledge/Understanding* and *Creative Application of Concepts.*

The three qualities of complexity, playfulness, and universal appeal were found to be germane to the integrated learning process. Furthermore, the following four elements were identified as key criteria for developing an integrated curriculum: creative space, adequate time, legitimate matter, and infectious energy.

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I gratefully acknowledge the participation of, and recommendations made by, the members of my examining committee at Nipissing University. I would also like to thank the many students of mathematics classroom 202, whose creativity and enthusiasm regarding the assignments made this project increasingly fascinating.

This study is dedicated to my wife April, and to our two small, curious daughters Clara and Anna; she, my safeplace, and they, my inspiration and joy.

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There is geometry in the humming of the strings. There is music in the spacing of the spheres.

Pythagoras (c.a. 582-500 BC)

Learning is the only thing the mind never exhausts, never fears, and never regrets.

Leonardo da Vinci (1452-1519)

Wonder is the salt of the earth.

Maurits Cornelius Escher (1898-1972)

Research Problem

The rigorous new *Ontario Curriculum Grades 9 and 10: Mathematics* (Ontario Ministry of Education and Training, 1999) requires evidence of *Communication* and the *Creative Application* of mathematics. The *Ontario Curriculum Grades 9 and 10: The Arts* (OMET, 1999) requires evidence of K*nowledge/Understanding* and T*hinking/Inquiry* skills within the visual arts program. The educational problem, which I perceive to currently exist, is that the majority of traditional teaching practices at the secondary school level do not facilitate high achievement of the above-mentioned expectations within these two disciplines respectively.

To ascertain the effects of an integrated curricular model on student perceptions and on achievement within the aforesaid categories, three integrated mathematics and visual arts assignments were created and implemented within an Ontario secondary school. The product of an arts-infusion approach within a mathematics curriculum framework, these three assignments were experienced by students in two Grade 9 mathematics classes as part of the regular course of study.

The remainder of chapter 1 features a contextual broadening of the research problem, followed by a rationale supporting an integrated mathematics and visual arts curriculum. It concludes with a presentation of the more specific research questions that have evolved with regard to the nature and scope of the research that was conducted in this study. It is my hope that this panoramic introduction to the research problem and research questions will better situate and inform the reader. It will also serve to facilitate any further related research.

The Problem and its Antecedents

This secondary school problem of limited student success in mathematics and visual art, regarding the aforementioned Achievement Chart categories, has had its roots in the elementary panel. Elementary teachers are predominantly, and by necessity, generalists who often have little, if any specialized knowledge in either mathematics or visual art.

Notwithstanding the recent trends in teacher education which encourage the use of a broad range of teaching strategies in the mathematics classroom, mathematics has often been poorly taught (National Council of Teachers of Mathematics, 2000, p. 5). In many mathematics classrooms, while basic algorithms and formulae were taught successfully, the higher order skills of reasoning, estimation, and problem-stating and solving were not being mastered (Paulos, 1988, p.73). Chatterley and Peck (1995) stated that this is often a consequence of “crippling students with kindness” as teachers with good intentions “exclude students from those experiences that come from struggling with a problem” (p. 435).

Likewise, visual art has often been denied proper materials, adequate space, qualified educators, and in many cases, a mere existence in the weekly elementary schedule. Morgan (1995) noted, “Art is sometimes viewed as a peripheral subject, requiring a subject specialist, and not of special significance in its own right. It can be easily ignored, relegated to the worst time slot, or treated as recreation” (p. 12).

It has been argued that artistic development proceeds in logical stages (Edwards, 1979, pp. 62-79). While teaching art in northern Thailand, I desired to put this bold hypothesis to the test. An invitation to all K-8 teachers at Chiang Mai International School was distributed, requesting that their students complete and submit three drawings under the following headings: (i) their family; (ii) a landscape (outdoor scene); and (iii) themselves doing some activity. The content of the over 500 artworks that were submitted, and the revelations during the interviews with selected students that ensued, both fully convinced me of Edwards' taxonomic accuracy, especially among children representing a wide variety of cultural backgrounds.

The stages that Edwards described culminate in the passionate quest for high realism that occurs at about age 10-12 years. Those who are fortunate enough to discover the art of *seeing* (the ability to render objects realistically) may continue in their art. The majority, however, will remain fixated on this failure and abandon artistic pursuits for a lifetime (1979, pp. 72-76). And so, besides the lack of artistic training among non-specialist staff, even art specialists are sometimes unaware of this natural developmental progression and are therefore unable to help students gain the needed skills and competencies.

Based primarily on my own limited pedagogical experience, I am of the opinion that a large number of Junior/Intermediate (Grades 5-8) students perceive mathematics as a painful, irrelevant subject meant to be endured or survived. In stark contrast, I have also noted that experiences in the visual arts are often used by teachers as a type of reward in weekly token economies. Is it any wonder, then, why these same students, while perhaps enjoying art class, perceive the visual arts as a tangent, trivial part of their education.

I do sympathize with the elementary teacher who is left to his or her own devices in magically conjuring a meaningful curriculum in disciplines for which he or she has not received adequate training. However, I am disheartened with these bitter realities since I strongly believe

that these two fascinating disciplines certainly have much more to offer at this stage of student development. And so to this effect, I have composed two kindred limericks that seek to epitomize my personal disappointment with what I perceive to be the traditional teaching of elementary school mathematics and visual arts.

MATHEMA ANATHEMA	**ARTIFEX AD ARTIFICE**
Mathema anathema, A phobia severe! That vile, loathsome topic, Which returns to haunt each year. Mathema anathema, The c(o)urse one loves to hate! Its shadow darkens every school; A nemesis of fate. Mathema anathema, Like words of ancient chant; Resounding through the centuries; A cry from all who 'can't.' Mathema anathema, Perhaps there be a cure; If math was simply relevant, And fun, one could be sure, that… Mathema anathema, Would all but disappear, As interesting lessons, Now dispel the former fear!	Artifex ad artifice, The wonder drug of art! That warm and fuzzy topic, Which caters to the heart. Artifex ad artifice, The course one hates to miss! Reward for good behaviour, Is mindless Friday bliss. Artifex ad artifice, 'Tis sophistry supreme; The students colour picture books, And leaf through magazines. Artifex ad artifice, Perhaps there be a cure; If art was simply relevant, And meaningful, then surely… Artifex ad artifice, Would all but disappear, As synthesis builds competence, For expectations clear.

Figure 1. *Mathema Anathema* (Jarvis, 1997); *Artifex ad Artifice* (Jarvis, 2000).

The Problem and its Progeny

The research problem also extends well beyond the secondary school. In fact, there exists an almost strange kind of pride amongst the masses regarding the inability to surpass both rudimentary child-like sketching and simple arithmetical calculations (Edwards, 1979, p. 2;

Griffiths, 2000, p. 1). Similar deficiencies in reading and writing skills would rarely be touted publicly in like manner.

A second initiative that I undertook while in Thailand was a professional development workshop/seminar that I offered to the international staff, entitled *Beyond Stickmen: Building Artistic Confidence and Awareness in the Classroom Teacher.* Approximately a dozen teachers participated in the two Saturday sessions, most of whom were elementary teachers. During preparatory drawings, several of the participants admitted a deep insecurity regarding their artistic skills, while a majority of the teachers present claimed to possess at least an undesirable ignorance of the discipline. Both a standard education *and* preparatory teacher training (K-8) from at least the five countries represented around the table (United States, Canada, England, Japan, and Thailand) had obviously not succeeded in building a sufficient visual arts foundation for practice.

Many adults suffer from what is known as math anxiety, or math phobia (Blomfield, 2000; Buxton, 1991), the symptoms of which likely went unnoticed, or were perhaps even adversely fostered, during elementary school. Paulos (1988) noted,

> The same people who can understand the subtlest emotional nuances in conversation, the most convoluted plots in literature, and the most intricate aspects of a legal case can't seem to grasp the most basic elements of a mathematical demonstration. They seem to have no mathematical frame of reference and no basic understandings on which to build. They're afraid. They've been intimidated by officious and sometimes sexist teachers and others who may themselves suffer from math anxiety. The infamous word problems terrify them, and they're convinced that they're dumb. (p. 88)

Upon completion of high school diploma requisites, many students never take another mathematics course. This is most unfortunate in at least two ways. First, they are then never privy to the culmination of all the different mathematics skills, having studied them for over a decade, that occurs in calculus, linear algebra, probability, statistics, and complex analysis (Fawcett & Cummins, 1970, p.370). Second, they are inadvertently denying themselves access to a large

number of jobs, not only in mathematics and computer science but in an increasing variety of fields which utilize mathematics (NCTM, 2000; Paulos, 1988).

The Problem and its Parameters

This research study has targeted the secondary panel, in which the new Ontario Curriculum (1999) has just been introduced. Like the provincial testing at the elementary level that has already transpired over the past few years, the Grade 9 and 10 curricula are driven by assessment. These new assessment policies are in turn driven by international paradigm shifts in both mathematics and visual arts education.

Mathematics. One of the largest and most prominent mathematics associations in the world, the National Council of Teachers of Mathematics [NCTM], released its latest educational treatise in April 2000 entitled Standards and Principles for School Mathematics. The following paragraph details several aspects of the problems that NCTM perceived to exist within mathematics education in North America:

> Evidence from a variety of sources makes it clear that many students are not learning the mathematics they need or are expected to learn (Kenney & Silver, 1997; Mullis et al., 1997, 1998; Beaton et al., 1996). The reasons for this deficiency are many: In some instances, students have not had the opportunity to learn important mathematics. In other instances, the curriculum offered to students does not engage them. Sometimes students lack a commitment to learning. The quality of mathematics teaching is highly variable. There is no question that the effectiveness of mathematics education in the United States and Canada can be improved substantially. (p. 5)

The Third National Mathematics Education Institute [TNMEI], a Canadian forum that in 1996 brought together top mathematics educators from Canadian universities across the country to develop a standardized vision of mathematics education for Canada, stated in one of its four resulting publications, *Tomorrow's Mathematics Classroom: Grades 7-9*, that,

> In tomorrow's mathematics classroom: mathematics is experienced as a diverse, powerful, and evolving discipline, as a way of thinking, as a way of communicating, and

> as a way of perceiving the world, with significant links to all aspects of human experience. (p. 6)

In July 1997, The Fields Institute for Research in Mathematical Sciences [FIRMS] and Northern Telecom [NORTEL] hosted an international workshop to provide a forum for discussion of the global and technological challenges facing secondary mathematics education. The resulting report, *Mathematics Education for the 21st Century*, made the following recommendations to the Ontario Ministry of Education and Training:

> To achieve the profile of the mathematics learner, it will be necessary to change fundamentally the way we implement mathematics education. The traditional transmission model, with students as passive learners, prevents students from acquiring the attitudes, knowledge and skills necessary for success in the twenty-first century. To be successful students must be engaged in the learning process, use technology when appropriate, and learn how to learn. They must acquire critical and creative thinking skills, abstraction and modeling capabilities earlier in their education. Mathematical concepts should be introduced within a context of inquiry and problem solving and should be organized around major concepts and themes. To measure the success and drive the changes, assessment must reflect the profile of the mathematics learner and the methods of instruction. (1997, p. 10)

Results from the Third International Mathematics and Science Study [TIMSS] were released in 1996 (Education Quality and Accountability Office [EQAO], 1996). Forty-six jurisdictions in 40 different countries were involved in this three-phase study which highlighted relative mathematics and science achievement around the globe. Roulet (2000, p. 15) noted that ranking from the TIMSS “can be combined with the performance measures to tell us about school systems, students’ lives, teachers, curricula, and instructional methods around the world.” Because Ontario scored relatively low on algebra and problem solving, he is concerned that although the new *Ontario Curriculum Grades 1-8: Mathematics* (Ontario Ministry of Education and Training, 1997) increases the algebra content, rich problem-solving has not been clearly mandated therein (p. 20).

A similar phenomenon is evident in the newly released *Ontario Curriculum Grades 9 and 10: Mathematics* (Ontario Ministry of Education and Training, 1999). Notwithstanding the theoretical emphasis placed on communication and problem-solving (pp. 3-4), the document has apparently left it to writing teams or to individual teachers to experiment with the *how* and *how often* of curriculum. As will be further delineated later in this paper, mathematics educators in Ontario have had to look elsewhere for inquiry-based models that enhance mathematics learning regarding the above-mentioned expectations.

The sentiments of Sir Wilfred Cockcroft, former chair of Britain's Committee of Inquiry into the Teaching of Mathematics in Schools, and author of the internationally acclaimed *Cockcroft Report* (1982), encapsulate the mathematical *zeitgeist* of our time:

> In saying this, I am also influenced by the strength of my belief that *at all levels* mathematics is about problem solving, so that abstract axiomatic systems only become worthy of study in their own right when we recognize the significance of the problem-solving mathematics from which they arise. (1994, p. 50)

In a manner resembling Hilbert's announcements of his mathematical prognostications and guiding questions delivered in Paris at the advent of the 20th century, Phillip Griffiths likewise addressed an international audience in Washington, DC, June 15, 1999 with his own predictions regarding the mathematics of the coming 21st century. Griffiths concluded his lecture by listing two main objectives for mathematicians: (i) to maintain traditional strengths in basic research, and (ii) to broaden exploration outside the traditional boundaries of the field – to the other sciences and the world beyond science (2000, pp. 13-14).

According to these sentiments of distinguished international authorities from within the discipline, mathematics education must evolve in both its content and context. As the foci shift from memorization to inquiry-based comprehension and from isolation to interdisciplinary communication, the case for an integrated curriculum is reinforced.

Visual Arts. As mathematics has undergone a paradigm shift towards a more subjective and varied assessment, the visual arts, in strong contrast, has experienced a shift towards objective and quantifiable assessment. The Discipline-Based Art Education [DBAE] initiative of theorist Elliot Eisner, has become, for most art specialists in Canada and the US, "the most widely promoted and adopted model for art curriculum over the last decade" (Irwin, 1993, p. 24). This theory champions the four essential categories of Art History, Art Criticism, Art Studio, & Aesthetic Discourse which comprise a quality visual arts program (Eisner, 1987). Assessment is a critical piece in the DBAE approach to art education. This would involve, among other elements: rubrics, peer- and self-evaluation, journals, critiques, exhibitions, and national standards.

Not only do art specialists and curriculum writers differ in their interpretations of *The Ontario Curriculum Grades 9 and 10: The Arts* (OMET, 1999) expectations, it is an unfortunate general truth that "art educators have traditionally been reluctant to endorse testing" (Jones, 1995). Jones maintained that teachers have feared that competition and the threat of failure will have an inhibiting influence on student performance in art (p. 34). He further commented,

> Just as it is easy to grade accurate computation in mathematics, it is easy to measure factual knowledge in art and the correct use of skills, such as perspective. Teachers, however, should resist pressures to teach only things that are easy to quantify. (p. 34)

Besides the lack of a benchmark for art teacher standards and professional development, Eisner further noted that public accountability is yet another critical piece in the defense of strong assessment practices for the arts. "The public expects public institutions to be responsible for the use of the funds it provides. . . . Protestations of faith and the expression of heartfelt commitment by art educators for art education will no longer do. The public wants evidence" (Cited in Boughton, Eisner, & Ligtvoet, 1996, p. 3).

In light of recent government initiatives and publications, there appears to be a positive future for art education in Ontario. Clark (1999) expressed his enthusiasm with regards to the recent trends:

> The current government of Mike Harris has affirmed the critical importance of art education in elementary schools with *The Ontario Curriculum, Grades 1-8: The Arts* (Ontario Ministry of Education and Training, 1998) in which both overall and specific expectations for each grade are specified along three curricular strands: knowledge of elements, creative work, and critical thinking. . . . The new secondary art courses will benefit from the more rigorous elementary preparation mandated by *The Ontario Curriculum, Grades 1-8: The Arts* and will operate within the protective programming umbrella of The Arts. (p. 50)

There exists a sense of international optimism regarding both the evolving mathematics and visual arts curricula, and as expressed by the cited proponents. The aforementioned changes in assessment paradigms within both disciplines are welcomed, yet not easily attainable. In the rationale that follows hereafter, I will attempt to demonstrate that an integrated mathematics and visual arts curriculum may have the potential to more effectively achieve these ends.

The Purpose of the Study

This research was conducted to examine the effects of three integrated Grade 9 mathematics and visual arts assignments on student learning and perceptions within three particular strands of the mathematics curriculum (*Number Sense and Algebra*, *Analytic Geometry*, and *Measurement and Geometry*). This research also sought to define qualities of the integrated learning process as evidenced through the various research instruments. These assignments were assessed as part of the regular Grade 9 academic mathematics course of study.

Rationale for the Study

In the following rationale I will attempt to justify an integrated mathematics and visual arts curriculum model by presenting relevant information from the following three areas: historical connections, pedagogical concerns, and current trends in education and the workplace.

Historical Connections

The Greeks. Mathematics and the visual arts have coexisted since the dawn of human history. Furthermore, beyond mere coexistence, they have been intricately interwoven through issues of form and function throughout every cultural era within that history (Jarvis & Jones, 1997). In ancient Greece, sculptors such as Phidias, designer of the Parthenon frieze panels and the monumental ivory and gold statue of Athena for which the temple was built, used the golden ratio extensively in his work. This mathematical proportion was believed by the Greeks to hold the profound secret of visual harmony in the universe. The legacy of this concept, which was later termed the divine proportion during the Renaissance, is still evident in both modern and post-modern styles of architecture and fine art. Alex Colville, one of Canada's most widely recognized artists and who frequently uses the golden section while engineering his compositions, described the aesthetic experience involved.

> Once you begin to perceive these relationships, circles, spirals, triangles and rectangles appear as if on their own. The beauty of it comes as a surprise, and its harmonies inspire joy. Part of the excitement is that these discoveries, though new to us, are about immutable laws that have been in force since time and space began. (Cited in Fry, 1994, p. 35)

Throughout history, art has made use of many other mathematical elements including "the geometric shapes, symmetry, the earth's measurements, the proportions of humankind, the patterns of the stars, conic sections, as well as the computer" (Attenborough, Pattison, Patsiatzis, & Muller, 1997). Newman and Boles (1992, p. xiv), authors of *Universal Patterns: The Golden Relationship of Art, Math, & Nature*, maintain that although these two disciplines are often viewed as polarities, they are in fact "the left and right hand of cultural advance: one is the realm of metaphor, the other, the realm of logic. . . . Our humanness depends upon a place for the fusion of fact and fancy, emotion and reason. Their union allows the human spirit freedom."

Socrates, Plato, and Aristotle represent a rich lineage of effective pedagogical transfer. All three of these teachers held to strong, but distinct, educational philosophies. Socrates, master of the question and answer method that bears his name, was convinced that education and healing were closely related, and therefore defined teaching as the "building, in a pupil, of a system of value priorities and preferences that defined the healthy soul" (Broudy & Palmer, 1965). Socrates was also one of the first Western educators to hold the belief that unless citizens had an understanding of art and music, they were not considered to be adequately educated (Naisbitt & Aburdene, 1990).

In his major work, *The Republic*, Plato described, using his metaphor of the divided line, four states of mind or ways of knowing, The lower levels of illusion (*eikasia*) and belief (*pistis*) encompassed the "distorted perceptions" found within poetry and art (1987, p. 316). To the third level of reason (*dianoia*) belonged mathematics and the sciences. "For Plato the study of mathematics was to shape the soul, just as music and literature shape the soul" (Broudy & Palmer, 1965, p. 44). Only at the fourth and highest level, known as intelligence (*noesis*), was the clearest mental vision and recognition of truth achieved through dialectic (pp. 41-42).

Whereas both Socrates and Plato acknowledged the important, yet separate and unequal, emphases on mathematics and the arts education, it was Aristotle, the most prized student of Plato's Academy, who is remembered for encouraging the cognitive "search for relationships between things apparently disconnected" (Newman & Boles, 1992). And so, it is this latter quest, most surely influenced in part by his predecessors and their teachings, that bears directly on an integrated approach to the two disciplines.

Finally, one would be remiss in discussing Grecian influence on interdisciplinary education without mentioning he who has been referred to as the father of both mathematics and music, Pythagoras. It was his passionate quest for learning in many different areas that inspired

his disciples to become members of his covert mathematical brotherhood, and to continue in his pursuits after his death.

Many of the philosophical, mathematical, and aesthetic pursuits of the ancient Greek culture would be revisited and expanded over a thousand years later during, what has become commonly known in modern retrospect as, the European *Renaissance*.

The Renaissance. The rebirth of culture and learning that pervaded Europe was intrinsically tied to the notion of the individual as brave and enlightened adventurer. This social metaphor had, of course, its literal parallels in the likes of explorers Columbus and Magellan who actually crossed unknown waters in search of the affluent East and national glory. But more fascinating still, was the emergence of the artist/scientist, bearing healthy traces of both neophyte and erudite simultaneously, who pushed the known boundaries of learning and discovery to the limits and beyond. De La Croix and Tansey (1986) highlighted this phenomenon:

> The wide versatility of many Renaissance artists – like Alberti, Brunelleschi, Leonardo da Vinci, and Michelangelo – led them to experimentation and to achievement in many of the arts and sciences and gave substance to that concept of the archetypal Renaissance genius – l'uomo universale, "the universal man." (p. 524)

One such man was the artist Raphael. In his large oil painting, *The School of Athens*, Raphael visually demonstrated the historical connections between the two disciplines by juxtaposing a number of great philosophers, artists and mathematicians of different eras. This was meant to symbolize the new and expansive learning of the Renaissance. Included in his academic fantasy scene were Plato, Aristotle, Pythagoras, Euclid, Leonardo da Vinci, his contemporary hero Michelangelo, and an unobtrusive self-portrait.

In 1983, IBM Canada Ltd. was involved with the development and sale of an educational packet entitled *I, Leonardo: A Journey of the Mind.* This extraordinary initiative included a video of historical reenactment, a filmstrip, audiotape, time charts, student handouts, suggested projects

and a teacher guide. Also in the kit was an introductory letter addressed, most unusually, to the "Science or Art Department Head," and which included the following paragraph:

> Most recent studies of Leonardo have continued the tradition of focusing on a single aspect of his multi-faceted character: Leonardo the artist, the scientist, the engineer, the city-planner, the architect. "I, Leonardo" attempts to place all these achievements within an interrelated framework; to explain how, for example, Leonardo's observational skills, developed through his work as a painter, extended his scientific insight, and how his scientific investigations enhanced his work as an artist. (1983)

This corporate endeavor not only highlighted the historical connections between mathematics and visual arts, but it served as a tangible example of how curriculum can be successfully integrated within an interdisciplinary context, using an historical character as a springboard for learning.

The Modern Era. The last two centuries have seen support for an integrated curriculum wax and wane. A well-rounded European liberal arts education in the 1800s would certainly have involved exposure to art and music as well as to the scientific disciplines (Naisbitt & Aburdene, 1990). One great controversy that arose in America during the twentieth century was that between the Progressives and another group of philosophers, including Dewey, that opposed the former group's methods. The Progressives in the 1920s established an integrated curriculum which was concerned with, among other things, the use of art to unlock the creative potential of children while teaching subjects from the other disciplines (Eisner, 1972). "The Progressives were committed to the natural development of the child, and the arts were viewed as an exceedingly useful vehicle for facilitating this development" (p. 48). Dewey, on the other hand, felt that art was a vehicle for developing marketable skills, learned in discrete steps. These theorists rallied against the notion of art being used to enhance other learning and promoted the idea that art should be treated as a separate discipline, the content of which should be determined by the art specialist or curriculum writer, not the individual teacher. The Progressive Education

Association eventually died out in light of the severe criticisms, but the argument at the heart of the debate over an integrated art curriculum has remained.

While some would argue that mathematics (or the sciences in general) has influenced the visual arts (Dorn, 1994; Golen, 1999), others would maintain that the preponderance of human experience shows the relationship to be vice versa (Shlain, 1991). In citing other theorists such as Whitehead, Laporte and Marti-Ibanez, Dorn (1994) built a strong case for the arts being heavily influenced by mathematics:

> In art, the twentieth century was a time when: (a) Einstein's discoveries of relativity and of the space-time continuum affected the way space was ordered in painting; (b) Marx's political thought and Freudian psychology radically influenced its form and content; and (c) the positivist ideas of Wittgenstein and the anthropological views of Levi Strauss inspired new uses of language and ritual now seen as the basis for the revisionist and the deconstructivist art of today. (p. 36)

Shlain (1991, p. 19) hypothesized that "repeatedly throughout history, the artist introduces symbols and icons that in retrospect prove to have been an avant-garde for the thought patterns of a scientific age not yet born." He supported this notion through examples from history, and interpreted the major metaphysical blurring between the disciplines as follows:

> While art is thought to be relatively subjective, physics, until this century, scrupulously avoided any mention of the inner thoughts that related to the outer world. Physics concerned itself instead with the objective arena of motion, things, and forces. This stark difference between art and physics blurs in light of the startling revelations put forth by the quantum physicists that emerged from the fusion of the contradictory aspects of light. . . . Thus "subjectivity," the anathema of all science (and the creative wellspring of all art) had to be admitted into the carefully defended citadel of classical physics. (p. 23)

Still other theorists such as Vitz and Glimcher (1984) have proposed a theory of Parallelism, in which the advances in both the sciences and the arts are often correlated through simultaneous expressions of perceived reality. They stated the following:

> These similarities cannot be treated as accidental because, as it will be shown, often the two works occur at about the same time, and frequently the artist's comments make it clear there was influence from visual science or that the artist on his own had discovered the same visual phenomena that contemporary scientists were investigating. Thus, it is

> argued that the artists' and scientists' parallel conceptual approach to vision frequently resulted in the construction of pictorially similar or even identical works. (p. 37)

Amidst a formal and consecutive presentation of such diverse theories, an integrated curriculum would remind students of the mutual influences that the disciplines have had on one another throughout history.

The connections between mathematics and visual art have been nowhere more apparent than in the visible works of art and architecture that combine the two disciplines in both physical and conceptual ways. A list of pertinent artists and architects from the last two centuries would include among many others: Escher, Stella, Calder, Moore, LeWit, Dali, Duchamp, Collins, Mondrian, Silverman, Verhoeff, Colville, and Pollock.

Throughout history, from ancient Greece to modern America, the disciplines of mathematics and visual art have been fused in theory, in education, and in the making of images and structures. An integrated curriculum would serve to reinforce these chronological connections as students experience an age-old approach to rich learning and application.

Pedagogical Concerns

Motivation. Having taught both disciplines at three different schools and in two very different countries, I can say without reservation that students in my visual art classes were usually more easily motivated, on average, than were those in my mathematics classes. If this is a consistent reality on a larger scale, then it would only make sense to capitalize on this phenomenon and use the natural enthusiasm children possess regarding art to help them gain a greater interest in the study of mathematics. Beane (1995), a leading proponent of an integrated curriculum, supported such an hypothesis in the following:

> We should ask that curriculum be kind to our young people, uplifting their hopes and their possibilities, instead of discouraging their spirits and aspirations. Its purpose should be to inspire our children, not to punish them. We should ask that it bring them joy in new insights and exciting discoveries. The work they do should involve more making and less

> doing, more building and creating, and less of the deadening drudgery that too many of our curriculum arrangements still demand. (p. 11)

Schramm emphasized the link between motivation and educational relevance in her research report regarding integrated mathematics and visual arts curriculum:

> The sense of 'connectedness' nurtured by an integrated curriculum should enable students to perceive the relevance of education to their lives. This heightened awareness of educational purpose should motivate students to become life-long learners – an essential characteristic of a modern citizen, enabling them to adapt to the demands of an unknown future. (1997, p. 5)

There also appears to be a high correlation between student motivation and classroom management. Jacobs, Yoshida, and Stigler (1997, p. 9) stated that "analyses of American and Japanese classrooms reveal that American lessons are less stimulating for children and encourage more off-task activities than Japanese lessons." Eisner suggested that, "Perhaps the effects – if effects there are – of arts courses on academic achievement are due to the motivational effects of arts courses; perhaps students in arts courses enjoy school more and therefore attend more regularly" (1999, p. 12). An integrated curriculum of mathematics and visual art would potentially serve to increase motivation amongst a higher percentage of students and may thereby decrease discipline problems in the interdisciplinary classroom.

Models of Teaching. An integrated curriculum allows for great freedom regarding the juxtaposition of various models of teaching. The creativity and problem solving that could be incorporated into the various unit projects and assignments would satisfy the higher levels of synthesis and evaluation from Bloom's taxonomy of educational objectives (Winzer, 1995, p. 544). Erickson noted, "Today, depth of instruction means teaching higher level, conceptual thinking by connecting ideas across disciplines to extend understanding, foster sound generalizations, and create new knowledge" (1995, p. 35).

Gardner’s theory of Multiple Intelligences (MI) has gained immense popularity amongst educational practitioners since its appearance in 1983. MI theory is most commonly applied to K-8 education. However, Armstrong made a strong case for its secondary school relevance:

> Children do not leave their multiple intelligences behind once they reach puberty. If anything, the intelligences become even more intense . . . Consequently, students should be learning their algebra, ancient history, government, chemistry, literature, and more through multiple intelligences. In algebra, students should be talking about the unknowns (the “x’s”) in their own lives. (1994, p. 27)

Other teaching models such as role play, inquiry-based learning, synectics, exhibitions, simulations, socratic dialogue, and computer-based learning (Joyce & Weil, 2000) could also be employed more easily within the flexible parameters of an extended integrated curriculum.

Brain Research. With the advent of modern computer technologies and advanced medical equipment, brain research experienced rapid development in the latter part of the twentieth century. Intricately tied to this body of ever-changing research and knowledge in the physical sciences, were the developments in educational psychology leading to changes in educational theory and practice (Miller, 2001; Sylwester, 1995).

One area of the research that has generated much conversation and controversy in the educational world is that of brain laterality, meaning that there are significant differences between left and right hemispheres of the brain. Edwards (1979) explained a popular definition of laterality from the 1960s and 1970s that had a significant effect on educational practice:

> For the past one-hundred fifty years or so, scientists have known that the function of language . . . is mainly located in the left hemispheres of the majority of individuals . . . this [knowledge] was largely derived from observations of the effects of brain injuries. . . . Because speech and language are so closely linked to thinking, reasoning, and the higher mental functions . . . nineteenth-century scientists named the left hemisphere the dominant or *major* hemisphere; the right brain, the subordinate or *minor* hemisphere. . . . Then during the 1960’s, extension of similar studies to human neurosurgical patients provided further information . . . and caused scientists to postulate a revised view . . . that both hemispheres are involved in higher cognitive functioning, with each half of the brain specialized in complementary fashion for different *modes* of thinking, both highly complex. . . . Most of our educational system has been designed to cultivate the verbal,

> rational, on-time left hemisphere, while half of the brain of every student is virtually neglected. (pp. 27, 29, 36)

Sylwester (1995) commented on the resulting educational reform, “We’ve already demonstrated our vulnerability with the educational spillover of the split-brain research: the right brain/left brain books, workshops, and curricular programs whose recommendations often went far beyond the research findings” (p. 6). He further commented on the more contemporary theories of laterality, “Although differentiation of tasks (lateralization) does occur, the hemispheres collaborate on most tasks, and the corpus callosum helps to synchronize their activities” (p. 49). Bamburg (1997) concurred with this notion, “In a healthy person, the two hemispheres are inextricably interactive whether a person is dealing with words, mathematics, music, or art” (paragraph 4).

Miller (2001) noted, “As teachers, we have an awesome responsibility to consider how brain research might shape our practice” (p. 1). Later in this same article, in which she reviewed current brain research as it relates to educational practice, she listed three vital characteristics of an enriched environment conducive to learning: (i) the environment must be stress free; (ii) the environment must stimulate the learner with open ended challenges; and (iii) the environment must provide ongoing feedback (p. 3).

An integrated visual arts and mathematics curriculum would seek to proactively facilitate a balanced exercising of both the synthetic and analytic aspects of brain laterality (Sylwester, 1995, p. 50), as well as to provide a challenging, stimulating, and stress free environment in which students experience creative projects and receive meaningful and ongoing assessment feedback.

Teacher Stimulation. To teach effectively within an integrated curriculum would require both expertise and flexibility; what Eisner recounted as fluid intelligence, or “intelligence in

progress” (1979, p. 161), in which new moves are artistically invented en route (1983, p. 11). I think that the potential stimulation and creativity generated by such a genus of learning would be extremely beneficial to the individual teacher’s wellbeing over time. Eisner defined the type of school environment in which this regular exploration could best occur:

> When one finds in schools a climate that makes it possible to take pride in one’s craft, when one has the permission to pursue what one’s educational imagination adumbrates, when one receives from students the kind of glow that says you have touched my life, satisfactions flow that exceed whatever it is that sabbaticals and vacations can provide. (1983, p. 12)

An integrated curriculum of mathematics and visual arts would not only allow for individual professional development as a teacher, but may also encourage team planning with colleagues, increasing the communication skills and the motivation of all those involved.

Current Trends in Education and the Workplace

North American Curriculum. Integration and interdisciplinary studies have experienced a groundswell of support in both the United States and Canada (Attenborough, Pattison, Patsiatzis, & Muller, 1997; Jardine, 1990; Schramm, 1997). As early as 1989 NCTM emphasized a focus on connections in their publication Curriculum and Evaluation Standards for School Mathematics:

> The mathematics curriculum should include investigation of the connections and interplay among various mathematical topics and their applications so that all students can . . . use and value the connections among mathematical topics; [and] use and value connections between mathematics and other disciplines. (1989, p. 146)

Throughout the past decade we have witnessed an increased number of references to an integrated program in the educational documents of Ontario. Both *The Common Curriculum* (OMET, 1995) and *The Ontario Curriculum* (OMET, 1999) present an awareness of the importance of seeking cognitive and physical connections between the disciplines.

> The ability to see links among different areas of learning will enable students to use the knowledge and skills developed in one field to learn in another and to relate their learning

> to real-life situations. . . . To encourage integrated learning, subject matter and outcomes are organized into broad program areas that include all the traditional subjects. Outcomes are designed both to emphasize the relationships among subjects and to focus on the knowledge, skills, and values related to a particular subject. Students thus have opportunities both for integrated and for subject-specific learning. (Common Curriculum: Policies and outcomes Grades 1-9, 1995, pp. 10-11)

Four years later, the same ideas were redefined in each of the newly released documents for each discipline. For example, the mathematics document featured the following statement:

> Other disciplines are a ready source of effective contexts for the study of mathematics. Rich problem-solving situations can be drawn from closely related disciplines, such as computer science, physics, or technology, as well as from subjects historically thought of as distant from mathematics, such as geography or art. It is important that these links between disciplines be carefully explored, analyzed, and discussed to emphasize for students the pervasiveness of mathematical knowledge and mathematical thinking in all subject areas. (The Ontario Curriculum Grades 9 and 10: Mathematics, 1999)

Parallel thoughts were reiterated in the visual arts document. “Links can also be made between the arts and other disciplines. For example, symmetry in musical structure can be related to mathematical principles. Mathematics skills can be applied to drafting a stage set to scale, or to budgeting an arts performance” (The Ontario Curriculum Grades 9 and 10: The Arts, 1999).

Furthermore, in every one of the discipline-specific booklets published in 1999, and again in the Grade 11 and 12 documents of 2000, the following paragraph was inserted into the body of the introduction.

> Subject matter from any course in mathematics can be combined with the subject matter from one or more courses in other disciplines to create an interdisciplinary course. The policies and procedures regarding the development of interdisciplinary courses are outlined in the interdisciplinary studies curriculum policy document. (The Ontario Curriculum Grades 9 and 10: Mathematics, 1999)

The resource to which it referred, the *Interdisciplinary Studies Policy Document*, could not be located. In an attempt to discover its whereabouts, the Ministry web site was searched and several phone calls were made. Apparently, as I was informed, the document was under

construction and was to be released in the near future. The document was not available by the time this research culminated in 2001.

Japanese Pedagogy. As part of TIMSS, several national organizations in the United States funded a special study (Stigler & Heibert, 1997) to further investigate the pedagogical practices in 231 Grade 8 classrooms in Germany, Japan and the United States. Roulet reported that “the picture of Japanese mathematics instruction that emerges from this study contrasts sharply with the typical image of Asian classrooms as places where pupils silently participate in endless drill and practice” (2000, p. 20). On the contrary, Japanese classrooms were found to be alive with rich problem-solving tasks, to encourage regular communication of mathematics ideas, and to facilitate deeper understanding of concepts (Jacobs, Yoshida, & Stigler, 1997; Roulet, 2000; Stigler & Hiebert, 1997). American classrooms were found to be much more directive, in which the teacher lectures on a new topic and then students practice problems quietly, and usually independently.

It is this conspicuous difference in pedagogy that has prompted mathematics educators and curriculum writers in the West to reassess traditional methodologies and entertain ideas of more inquiry-based, cooperative mathematics learning environments. An integrated curriculum would allow for this type of learning, in which students would be encouraged to individually research mathematical phenomena, to cooperatively solve problems of mathematical inquiry, and to communicate their mathematics ideas in both verbal and written forms.

A Changing Workplace. As Huitt has noted, “We are undergoing the most significant change ever experienced in human history. We have moved from the agricultural age through the industrial age and into the information age in a span of just 100 to 200 years” (1997, paragraph 8). This cultural metamorphosis has brought with it a demand for new skill sets, both specialized and generic. The former is often intrinsically tied to the computer revolution, and the latter set,

according to *Canada Prospects 1999-2000*, includes academic, personal management, and teamwork skills (Human Resources Development Canada, 1999, Section 5, p. 3). Daniel Goleman's theory of Emotional Intelligence (O'Neil, 1996) has replaced both IQ and strategic management skills in becoming a popular contemporary focus of the business world. These prized skills involving self-knowledge, confidence, and discipline will be highly sought after in the new economy.

Notwithstanding the absolute necessity of basic skills competency in both visual arts and mathematics, digital literacy has become another key factor across the curriculum. Already, the new Grade 9 courses have demanded secondary teachers to become familiar with a variety of technologies. In mathematics, I have incorporated into the learning, among other technologies: TI-83+ graphing calculators, motion detectors, databanks, spreadsheets, word processors, the Internet, and *Geometer's Sketchpad.*

This last item, a computer software package licensed by the Ontario Ministry of Education and Training, provides a complex link between the two disciplines in focus. Using this software, students are able to enter numerical data for assigned problems and then to analyze dynamic (e.g. models that alter according to changes made in the numerical data) and colourful mathematical models and corresponding graphs, which appear simultaneously on the computer screen. This is an entirely new style of learning for students through rich explorations, and this type of software may very well revolutionize how many of the topics, especially geometry, are presented from K-12. Along with graphic visual art programs such as *CorelDraw*, *Adobe PhotoShop*, *Quark Express*, and web publishing software, this *Geometer's Sketchpad* epitomizes the marriage of mathematics and visual art skills within the computer realm.

Cockcroft (1994, p. 46) advised that "students should be introduced as early as possible to the use of software packages, both content-specific (e.g. for developing number bonds, exploring

properties of the triangle, solving logical problems in adventure games, etc.) and general-purpose (e.g. spread-sheets, database packages, graphics packages, etc.).” Bergland (1996, p. 152) asserted the following, “Within this vigorous image revolution, art education has the opportunity to assume its place as rightful leader in the educational advancement of the new technologies which will comprise the working heart of visual culture in the 21st century.”

Interpersonal skills, critical thinking, and flexibility will also figure largely in the new economy. “Clearly, with knowledge constantly growing and changing in all areas of study, it is important for educators to prepare young people to be adaptable thinkers, researchers, and problem solvers” (Schramm, 1997, p. 5). Whereas mathematics has traditionally focused on logical proof and analytical prowess, the visual arts have always highlighted creativity and personal expression. Were these subjects to be regularly fused, the complementary skill sets that may potentially emerge in greater strength would certainly serve our student populations well in light of the pressing employment prerequisites.

In conclusion to this rationale, I quote Fogarty and Stoehr (1995), authorities on curricular integration, who summarized their interpretation of the changing educational climate and the role that they perceive integrated learning will play in the future:

> The winds of change are stronger than we think. The brain research, the unloading of an overloaded curriculum, the necessity for the life skills of thinking and cooperating, and the call for learner-centered schools are all forces that are moving educators toward integrated, holistic, and authentic learning. (p. 2)

Research Questions

Research Question

Notwithstanding the broader scope of the above rationale supporting a fully integrated curriculum, the following research questions will deal specifically with the arts-infusion approach that was used, by necessity, within this research study.

The principal question of this research is the following: What affect does the implementation of three integrated, mathematics and visual arts assignments have on student perceptions and achievement in the mathematics classroom? This question is further subdivided into five supplementary questions.

Supplementary Questions of the Research

1. Are the segments of the integrated curriculum pertaining to the *Communication of Mathematics* perceived by students as having reinforced the new mathematics learning?

2. Are the segments of the integrated curriculum pertaining to the *Creative Application of Mathematics* perceived by students as having reinforced the new mathematics learning?

3. Are the integrated assignments perceived by students as having had any effect on student motivation in the course?

4. What are some unique qualities of the integrated learning process that become evident through the creation of the assignments and the analysis of the other research instruments?

5. What patterns emerge from the assessment of student work, regarding the various Achievement Chart categories?

Definition of Terms

Curriculum

Curriculum is traditionally defined as all the courses of study offered by an educational institution, or as a group of related courses, often in a special field of study (American Heritage Dictionary [AHD], 1995). For the purpose of this study, curriculum will be discussed in terms of all the student activities directly related to the knowledge, skills, and attitudes that an integrated curriculum may foster. These need not be confined to the classroom proper, and may include field trips, project research, exhibitions, etc.

Integration

Integration, simply defined, is to make into a whole by bringing all parts together; to unify or to join with something else (AHD, 1995).

> Integrative studies are typically based upon the notion that boundaries are not evident, or at least not readily evident, and that relatedness of concepts, thoughts, feelings, and ideas emerge and grow among experiences. Therefore, the concept of integrative studies is broader and more inclusive than that of interdisciplinary studies. (Irwin & Reynolds, 1995, p. 15)

Although this distinction is helpful in understanding the literature, I will use the term integration to refer to the assignments experienced in this study, even though it will only directly involve two traditional subject disciplines, and therefore would likely be defined by some as interdisciplinary, and not truly integrated.

Interdisciplinary

The term interdisciplinary means relating to, or involving two or more academic disciplines that are usually considered distinct (AHD, 1995). "Interdisciplinary studies share a belief in relatedness among experiences, while recognizing that each discipline is a discrete body of knowledge with boundaries and should be examined as such" (Irwin & Reynolds, 1995, p. 15).

Line

Line defies simple definition. The American Heritage Dictionary (1995) lists 35 working definitions from a wide variety of professions, traditions, and vernacular expressions. The first definition listed is this – the path traced by a moving point. This will suffice as a foundational image, bearing in mind that many of the remaining denotations were explored throughout the integrated learning. The title of this study, *Learning Between the Lines*, sought to communicate two key elements of the research: (i) that, as in the common phrase "reading between the lines," a higher level of thinking is required to ascertain truth, so too an integrated curriculum seeks underlying principles and elemental connections; and (ii) that the Ontario Grade 9 curriculum for

both mathematics and visual arts is intrinsically tied to the concept and construction of the line in two- and three-dimensional space. As Dewey (1934) noted in his work, *Art as Experience*, “Line relates, connects. It is an integral means of determining rhythm” (p. 202).

Mathematics

Mathematics is defined as the study of the measurement, properties, and relationships of quantities, using numbers and symbols (AHD, 1995). As a discipline it has traditionally included topics such as arithmetic, geometry, algebra, analytic geometry, probability, and the calculus.

Syncretism

Syncretism is the reconciliation or fusion of differing systems of belief, as in philosophy or religion, especially when success is partial or the result is heterogeneous (AHD, 1995). This term has been applied to the title of this research experiment due to: (i) the traditionally acute polarity that is popularly understood to exist between the disciplines of mathematics and visual arts; and (ii) the realistically projected imperfection of this philosophical and practical fusion.

Visual Arts

The visual arts denote the discipline in which art, produced or intended primarily for beauty rather than utility, is skillfully created through a variety of forms including drawing, painting, sculpture, print-making, etc. (AHD, 1995). The visual arts are to be distinguished from the *Arts* in general, which often denote the entire family of subjects within the liberal arts, and from the sciences. It should also be contrasted with the *Fine Arts* which, though more specific than the *Arts*, would also encompass music, dance, and drama.

Chapter Two: Context and Review of the Literature

Theoretical Framework

"The debate over subject-centered versus interdisciplinary education enjoys a long history" (Woodbury, 1998, p. 303). I believe that this controversial polemic can be best analyzed according to two key questions.

Question 1: Should the Disciplines Exist as Organizers?

The Balm of the Disciplines. Vindicators of a discipline-based curriculum are myriad. Its longstanding existence as a systematic form of educational organization speaks to its utility. Mason (1996) stated:

> Scholars have historically organized and taught human knowledge in the context of the major disciplines of mathematics, philosophy, the sciences, the arts, and the humanities and the subject matter areas that have been built around them (e.g., social studies). Many consider the academic disciplines a useful framework for organizing curricular content because they constitute the most sophisticated ways yet developed for thinking about and investigating issues that have long fascinated and perplexed thoughtful individuals . . . they become, when used relevantly, our keenest lenses on the world (Gardner & Boix-Mansilla, 1994, p. 16-17). . . . Introducing students to the forms of discourse and inquiry associated with the disciplines and the subject matter areas can provide them with valuable mechanisms for making sense out of a complex world. (p. 263)

Interestingly, in a similar in-depth analysis, Case (1994) noted that the disciplines, in their very structure and essence, bear a form of integration themselves. He stated that "the disciplines, for all of their narrowness, provide integrative principles – disciplines are fields of inquiry that share standards of evidence, fundamental explanatory concepts, and methodological procedures" (p. 86). Gardner (2000) explained the history and nature of the disciplines at length. As the creator of Multiple Intelligences Theory, which has been widely used by integration theorists, Gardner is somewhat ironically a devout advocate of a discipline-based curriculum:

> Disciplines have endured over the ages, even as their identities and boundaries have shifted. At any given moment, the disciplines represent the most well-honed efforts of

> human beings to approach questions and concerns of importance in a systematic and reliable way. (p. 144)

The Bane of the Disciplines. I suppose that if the disciplines could be likened unto nuclear fission in which particles separate, releasing power, sometimes devastating; then an integrated model might be seen as nuclear fusion wherein particles are forced together, releasing exponentially more power, yet dangerously uncontrollable. The debate forcefully continues.

Many proponents of an integrated curriculum have rallied bitterly against the discipline model. In the rationale of their integration document, Pattison and Patsiatzis explained:

> In the last decade, the education system at the secondary level has attempted to create students who would be experts in specific fields of study through the concept of delivering strictly specialized courses and in some cases creating specialized school buildings. This resulted in overspecialization where students acquired "knowledge" as disjointed bits of information. (Attenborough, Pattison, Patsiatzis, & Muller, 1997, p. 2)

Mason claimed that, "The traditional curriculum fails to meet the needs of students in a complex, technologically advanced, interdependent world" (1996, p. 265). This concern has been reiterated by Schramm (1997):

> The traditional subject-centered curriculum does not reflect the social reality that our students experience in their everyday lives. According to Jacobs (1991), the discipline-oriented structure of traditional curricula has contributed to the current crisis in American schools. Many young people are physically dropping out, while others drop out mentally, sitting passively in their classrooms waiting to survive the day. (p. 5)

Fogarty (1991), in describing her first of ten types of integrated models, criticized the fragmentation that she felt plagues the discipline approach to education. "In middle and secondary schools, the disciplines are taught by different teachers in different locations, with students moving from room to room. Each separate encounter has a distinct cellular organization, leaving students with a fragmented view of the curriculum" (p. 62). But perhaps the most severe denouncement of the discipline model came from Brady (1993):

> The academic disciplines began as means, and they have become ends. Now, in most classrooms, the reality the disciplines were designed to illuminate is of less consequence

> than the history, vocabulary, methodology, and procedures of the disciplines themselves. . . . Despite their nearly universal use as organizers of education, the traditional disciplines are not the best available tools for teaching about reality. If we looked at them critically, we would see that they are poor material from which to build a general education curriculum. They ignore vast and important areas of knowledge. They give instruction no overarching aim. They have nothing to say about the relative importance of various kinds of knowledge. They do not give students a mental framework for organizing and relating what they are taught. Because they cannot be made to integrate with one another, they fail to disclose the systemic nature of reality. . . . In fact, "discipline-based general education" is an oxymoron. (p. 439)

Question 2: How Should Integration be Defined and Implemented?

Caveats. Both mathematics and visual arts educators have expressed their valid concerns regarding the defense of their discipline's integrity within interdisciplinary curricula. For example, I can recall a university mathematics education professor sharing his consternation regarding the ridiculously simplistic mathematics skills required for, what he described as, nefariously integrated projects at the secondary level. A (negative) reciprocal anxiety also exists among art educators.

Eisner (1999) presented three categories of arts outcomes: arts-based, arts-related, and ancillary. It is to the latter type that he relegated the use of art in boosting academic achievement in other disciplines. Although he did not oppose an integrated curriculum, he strongly maintained that,

> We do the arts no service when we try to make their case by touting their contributions to other fields. When such contributions become priorities the arts become handmaidens to ends that are not distinctively artistic and in the process undermine the value of art's unique contributions to the education of the young. (p. 149)

At the outset of the same article, Eisner had inquired in jest, "Have they [theorists] ever thought about asking how reading and math courses contribute to higher performance in the arts? I must confess I have never come right out and asked them – but I have come close" (p. 8).

Other visual arts specialists have articulated similar concerns. Cormack (1991) sardonically explained, "Cranked-out rabbits and turkeys are not art education, listening to

records as a time killer is not music education, and being a pilgrim does not a Thespian make" (p. 42). Grauer also noted,

> Illustrating a story, the title page for a social project, colouring in the geometric shapes after a math lesson, are all examples of this type of rather inane "art" experience. . . . There is no substantive art learning taking place, rather art is used as a motivator at best and a time filler at worst. Again, we come back to the need to understand the discipline of art before an integrated experience can have educational goals. (1991, p. 26)

Clark likewise warned, "Whenever art education is delivered through an integrated format, care must be taken to ensure that it operates as an equal partner in a nurturing curricular environment" (1995a, p. 39).

The second area of caution that became evident in the literature review was the various problems associated with the implementation of integrated models. For some educators, integration has simply meant license for disorganization and a smokescreen for lack of planning. Marshall provided this disclaimer, "In the chaotic conditions created by misunderstood 'individualism' and 'unlimited freedom,' integration of anything can change rapidly into fragmentation of everything" (1970, p. 14).

Other poignant issues that arise are the following: lack of planning time, forced connections regarding themes, lack of teacher expertise, training, and approval, student disinterest during extended projects, lack of relevant materials, and the abandon of discrete instruction in basic skills (Barth, 1995; Case, 1994; Hanley, 1994; Woodbury, 1998). In light of these multiple barriers and caveats, I appreciate Mason's (1996) pertinent conclusion:

> The works of artists, scientists, and other creative individuals who have advanced knowledge and culture by transcending the boundaries of the disciplines attest to the value of striving for an integrated curriculum. The efforts of those who have examined the practice of curriculum integration over the last decade can provide a foundation for educators seeking to establish interdisciplinary programs in schools. Only by proceeding with caution, however, and by attending to the potential obstacles identified here, can educators achieve an integrated curriculum that will result in a better curriculum. (p. 271)

Models of Implementation. As Irwin and Reynolds have noted, "A first step on the path to clarification is for educators and academics to recognize that there is no consensus about the meaning of integration" (1995, p. 15). Germane to this elusive definition is the plethora of integration models that have been proposed. They have included specifically a survival-based model (Brady, 1993), an ecology-based model (Jardine, 1990), a self-disclosure model (Clark, 1991), a problem-based model (Pring in Irwin & Reynolds, 1995), and a theme-based model (Jacobs, 1989). In more general terms, theorists such as Fogarty (1991), Badley (Cited in Irwin & Reynolds, 1995), and Woodbury (1998) have presented categorical analyses of a variety of models, often in a continuum format.

One recurring approach is the grouping of the disciplines into broad areas of experience, as has been done with the arts in the new *Ontario Curriculum* (OMET, 1999). However, even this process creates tension regarding the lines that are drawn. Clark noted,

> While all four arts subjects are unique and of equal educational value, given what we know from current research into multiple intelligences (Gardner 1983, 1989), it is important to acknowledge that art usually stands apart from the performing arts, which include dance, drama, and music. (1995b, p. 3)

In fact, visual arts may be more suitably paired with English, both topics emphasizing creativity, or even with mathematics, for multiple reasons heretofore detailed.

Bickley-Green (1995), in her thorough paper entitled, "Math and Art Curriculum Integration: A Post-Modern Foundation," concluded with the following statement regarding mathematics and visual art integration:

> The study of integrated math and art in curricula shows potential for congruent activities that will enhance learning in both disciplines. Educators can do further research to demonstrate how art and geometry are cognate cognitive structures; teachers of math and art can work together to identify and instruct developmentally appropriate and integrated math and art activities for learners; and curriculum writers can reform the two courses of study by considering the related webs of meaning between the disciplines before creating impenetrable boundaries. (p. 17)

Related Integration Studies

There exist opposite conclusions regarding the effects of arts integration on student achievement. According to Gardiner, Fox, Knowles and Jeffrey (1996), research in two Pawtucket, Rhode Island elementary schools "produced strong evidence that sequential, skill-building instruction in art and music integrated with the rest of the curriculum can greatly improve children's performance in reading and math" (p. 8). Attenborough, Pattison, Patsiatzis, and Muller, quoting from the OSSTF Forum, similarly stated, "At the high school level worldwide, studies have shown that college and university bound students who combined visual art with mathematics in an integrated format had significantly higher entrance scores than students who studied math without studying art" (1997, p. 2).

Despite the occasional success story that surfaces, curricular analysts such as Mason (1996) and Case (1994) have painted a very different picture of integration studies. The former gave this advice, "For those who believe that valid research evidence is necessary to demonstrate the worth of educational practices, the results regarding curriculum integration offer little support" (Mason, 1996, p. 267). Case noted the possible negative results of integration:

> When we embark on educational reform we experiment with students' welfare, subject great numbers of teachers (and parents) to considerable additional work and anxiety, and spend increasingly scarce educational dollars. Furthermore, each failed attempt diminishes our system's capacity to effect change by destroying the trust and support of those who must shoulder the burden of future reform efforts. (1994, p. 89)

Eisner (1999, p. 10), in referring to 19 analyzed studies, concluded, ". . . the differences were statistically non-significant and, in my view, educationally trivial." Obviously there exists among education theorists, quite polar viewpoints regarding the validity of an arts-infusion approach to curricular integration.

Several research studies were found which resemble, to a greater or lesser degree, my research study. Schulte (1984), Omniewski (1999), and Willett (1992) found that by using an

arts-infusion approach with control groups at the elementary level, significant gains were witnessed in visual motor skills, retention of mathematical concepts, and mathematical achievement respectively. Borrelli (1987) showed that inservice training with a focus on integrated curriculum greatly improved the use and success rate of an integrated model with elementary school teachers involved. Through surveys and interviews, Alesandrini (2000) found that teachers expressed a positive view of integrated curriculum, provided that relevant materials, time and money were made available.

Biller (1994) had students state, illustrate, and solve a problem to facilitate understanding. He claimed that by drawing the picture, students “were able to put the concept into perspective (a schema), and that this resulted in increased comprehension and decreased math anxiety” (p. 4). Wersan (1981) and Schramm (1997) conducted the two formal studies that most closely parallel my research. Both were done at the secondary level and featured an interdisciplinary model using mathematics and visual arts. Wersan designed a two-week study in which he required one group to use a student-generated visual art approach to learning how to solve proportion word problems. Then through a series of pretests, posttests and delayed posttests he concluded that a student with high scores in “surface development” would achieve greater success in the art treatment method (Wersen, 1981).

Schramm (1997) took a much more qualitative approach and focused on student perceptions during an integrated math/art unit regarding the construction of greeting cards. Her research tools were surveys and semi-structured interviews. She concluded that student perceptions regarding an integrated curriculum were greatly changed from the conception of the project through to its completion.

One further publication that incorporated the disciplines of mathematics and visual arts in a practical way for secondary students is, *Mathematics and Visual Art: An Integrated Programme*

for Junior, Intermediate and Secondary Levels Inspired by the Work of M.C. Escher by authors Attenborough, Pattison, Patsiatzis, and Muller, (1997). Ideas similar to those presented in the above document were put into actual practice by Doctorow (1997) and Parkinson, mathematics and visual art teachers respectively.

Under their guidance, seven Grade 11 and Ontario Academic Credit (OAC) students volunteered to complete their required independent study mathematics projects by analyzing, planning, and creating a linocut print based on Escher's tessellations. These seven students participated in 15 specially arranged classes with the printmaking art class and maintained a portfolio which included all their preparatory creations, a journal with reflections on their work, and an analytic writeup of their Escher-style project. Both the mathematics and visual art components contributed to their overall mark, and students who were taking both the visual art and mathematics classes received credit in both subjects. Doctorow concluded their study with the following remarks:

> Our aims in doing this combined project work were to: provide as meaningful, creative, and interesting cross-disciplinary work as possible; provide an opportunity for creative and meaningful professional development; and advance curriculum and teaching at our school. From the enthusiastic responses of the students and the satisfaction both teachers felt, the goals were achieved. (1997, p. 21)

Action Research

Although the original letters of information and consent for this study (see Appendices) refer to Glaser and Strauss' Grounded Theory, I have chosen to subscribe to an Action Research model for qualitative case study, in that I believe it better coincides with the nature and purposes of the research.

Action Research possesses a relatively long and broad history, and has recently enjoyed a resurgence in popularity within educational practice. "It is becoming recognized as an invaluable tool for professional growth in Ontario and around the world" (Delong & Wideman, 1998, p. 11).

In an historical background sketch of the movement, McKernan (1996) noted, “Action research has attempted to render the problematic social world understandable as well as to improve the quality of life in social settings. Action research has been used in industrial, health, educational, and community behavioural settings” (p. 3).

Although the precise definitions of Action Research are myriad, they all tend towards the same key principles. Mills identified the process in the following way,

> Action research is any systematic inquiry conducted by teacher researchers, principals, school counselors, or other stakeholders in the teaching/learning environment, to gather information about the ways that their particular schools operate, how they teach, and how well their students learn. (2000, p. 6)

In a more personal treatment, McNiff defined Action Research as “a way of working that helps us to identify the things we believe in and then work systematically to make them come true” (Cited in Delong & Wideman, 1998, p. 16). Delong and Wideman described it as a system where “teachers use their actual classroom practice to improve their teaching and students’ learning and to generate their own living theories of education” (1998, p. 11).

The steps involved in, or the key components of, Action Research have also been listed in a variety of different, yet similar ways. McNiff (1993, p. 7) noted Whitehead’s outline as follows,

1. Identify a problem when some of my educational values are denied in my practice;
2. Imagine a solution to the problem;
3. Implement the solution;
4. Evaluate the solution;
5. Modify my ideas and my practice in the light of the evaluation.

Other variations on this theme would include Delong and Wideman’s description of “action, reflection, revision, and revitalization” (1998, p. 11), and Mills’ list featuring the “identification of an area of focus, a collection of data, an analyzing and interpreting of the data, and the

development of an action plan" (2000, p. 6). Or again, in perhaps the simplest terms possible: see, do, think, and react.

This research study involved a focus on integrated learning in mathematics and the visual arts. Three assignments were created and implemented. Data were collected in a variety of qualitative forms, analyzed, and interpreted. Conclusions were drawn, the original assignments were modified and improved, and future research possibilities were then presented. In these ways, I have experienced the phenomenon known as Action Research and have benefited from this experience both personally and professionally. The remainder of this thesis will describe this journey in detail as I attempt, in the words of Whitehead, to "show, in my living educational theory that I am bringing more fully into the world, those values, skills and understandings which I believe can contribute to a more peaceful and productive world" (Cited in Delong & Wideman, 1998, pp. 115-116).

Chapter Three: Methodology and Procedures

Research Design

The concept of an integrated curriculum is certainly not unfamiliar in education, or even in my own practice as an educator. I have often used my artistic training (Honour Specialist in Visual Arts) to introduce mathematics topics, as well as using my mathematics training to occasionally guide artistic instruction.

Although the ideal circumstances for this study would have involved a fully integrated Grade 9 mathematics and visual arts course, this ideal study was not practical or feasible. Therefore, an arts-infusion case study approach was used. Three integrated assignments, each focusing on a key topic from the 1999 Ontario Curriculum (proportion, linear relations, and geometric patterning), were given to two Grade 9 academic mathematics classes. Upon their respective completion, the three final projects were presented by students to the class. These presentations were video-taped for further analysis. Qualifying students, those with parent/guardian approval, completed questionnaires and a survey regarding the experiences, and certain randomly selected students were interviewed individually, and in groups, towards the end of the semester. An Open House was hosted at the culmination of the research, in order to allow parents/guardians an opportunity to view the exhibition of student work (written and visual) and to discuss the integrated assignments that their children had experienced.

As the teacher and researcher, I was in a unique position as participant-observer to be directly involved in the learning process and to make a careful record of daily observations and reflections in a field notebook. Fraenkel and Wallen (2000) described qualitative investigators as those who “wish to obtain a more complete picture of the educational process” (p. 12) and as those who view hypotheses as elements that “emerge from the data as the study progresses” (p.

505). By using a variety of research instruments, both qualitative and quantitative in nature, the conclusions of this study are strengthened via triangulation (p. 506).

Furthermore, this notion of dynamic hypotheses was certainly reinforced by the evolving nature of the assignments, the assessment, the questionnaire, the interview format, and the supplementary research questions themselves.

Research Sample

The participants in this research study were a convenience sample randomly assigned to me through normal timetabling procedures. Although all students were required to complete and present the three assignments as part of the regular course of study, only those who returned letters of informed consent, to be signed by both the parent/guardian and the student, were able to participate in the questionnaires and possible interviews. Of the 52 students registered in the two Grade 9 academic mathematics courses, 47 returned signed letters of consent allowing them to participate. Of these 47 students, 33 returned forms that further permitted the researcher to video-tape their classroom presentations of the SMARTWORKS (Secondary Math and Art Works).

The 47 students participating consisted of 19 females and 28 males who displayed a typical range of mathematical ability in areas such as performing algorithms, problem-solving, applying concepts, and communicating mathematics understanding that any regular class at the academic level would be expected to demonstrate.

Ethical Considerations

This research study proposal was approved by the Ethics Committee of Nipissing University, North Bay, Ontario in the summer of 2000. All subsequent changes or additions to the study were also sent to the committee chair and were approved before proceeding. The research was also approved by the local board of education, the administration of the secondary

school, and the department head of mathematics, each receiving an informative letter outlining the proposed study (see Appendices A, B, and C).

A Letter of Consent to Parent/Guardian and Student (see Appendix D), explaining the research and requesting permission for students to participate was sent home in September 2000, prior to the beginning of the assignments. This letter also explained how student participation in the various research instruments would in no way affect their achievement in the course. Likewise, parents/guardians were again reassured that their own comments, made either verbally at the Open House or submitted in writing, would not affect the success of their child in the course. Student work and comments were coded, thereby guaranteeing anonymity throughout and beyond the research process.

Pilot Study

The pilot study for this research was conducted during the spring term of 2000 in my Grade 11 Advanced mathematics class of ten females and eight males. One of these was an adult student completing both selected secondary and university courses simultaneously, en route to medical school. This added yet another interesting perspective to the project. Similar in nature to the integrated Grade 9 version that would follow, the assignment for the Grade 11 students also focused on the golden section but was much less developed, with no accompanying assessment rubric and no direct connections to government curriculum. Students were required to read an article by Griffiths (2000), entitled "Mathematics at the Turn of the Century," as a springboard for ideas. They were then instructed to create an art/math project that incorporated both the golden section (see Appendix E for details on the golden section/ratio) and the element from the article that had intrigued them the most (see Appendix O, CD-ROM, for Grade 11 student work).

This assignment included both a small, written description of their final product and a short classroom presentation of the same. The projects were exhibited in the classroom for the

remainder of semester. Students were asked to sign a permission form, allowing the potential use of any comments or assignments as part of any future research, and ensuring confidentiality. They were further asked to complete a questionnaire similar to one developed by Schramm (1997). Finally, 12 of these 18 students were available and agreed to be interviewed over eight months later, just prior to the Grade 9 interviews, using a similar interview format and content (see Appendix K). This time-lapse element permitted me to gain valuable insight into their perceptions of the integrated assignment, without any of the perceived pressure regarding student achievement that may possibly have accompanied the Grade 9 interviews.

Results from the Grade 11 questionnaires and the delayed interviews will be dealt with in more detail in chapter 5 alongside those from the Grade 9 research study. Suffice it to say that by the time the pilot study had drawn to a close I was very much impressed with both the creativity of the students and the general nature of the integrated learning and its various qualities. There was definitely something here worth exploring in much fuller detail.

After experiencing considerable anxiety regarding the possible outcome of this unique form of assignment, I now was more relaxed, and even excited. I was learning, as Mills noted in his discussion of postmodernism and critical action research, that "action research conducted in your *own* classroom/school is more likely to be persuasive, relevant, and the findings expressed in ways that are meaningful for you and your colleagues" (2000, p. 8). I was ready for the next step.

Instrumentation

Data for this research were collected using the following seven instruments:

- Field Notes
- Three Integrated Assignments
- Video-taped Presentations

- Student Questionnaires
- Final Student Survey/Questionnaire
- Interviews
- Response Form & Open House for Parents/Guardians

Field Notes

A diary/log/sketchbook was kept to record my observations and reflections for the duration of the five months over which the three assignments occurred. It included a variety of entries such as curricular ideas and revisions, student/teacher conversations, evolving hypotheses, areas of concern, humourous anecdotes, and the occasional sketch or illustration.

Integrated Assignments

The *Learning Between the Lines* research study was presented verbally to the students, and visually on a central bulletin board (see Appendix O), as a journey with three historical destinations throughout the term: the first in ancient Greece, the second in High Renaissance Italy, and the final in twentieth century Europe.

The three assignments were generally introduced and completed in the following way: (i) assignments were handed out, with supplementary pages of background information, and read over together as a class, (ii) a relevant video, books, and student samples were shown to create initial interest, (iii) a visual arts lesson was taught encompassing needed skills, (iv) a week later, a cooperative groupwork session was experienced, and (v) after approximately one month the assignments were submitted, the SMARTWORKS being presented to the class and then later exhibited in the classroom.

The first assignment, entitled *Ratio and Proportion: The Golden Section* (see Appendix E), was introduced with the video *Donald Duck in Mathemagic Land* (Disney, 1960) which highlighted many mathematical connections to music, art, architecture, sports, games, and nature;

as well as describing in detail the golden section and its significant place in history. Students were taught how to construct a golden section and rectangle using both geometric and algebraic methods. They were each given a large white poster board with which they were to construct the final SMARTWORK, while being assured that any other form or choice of materials would be acceptable and encouraged. Particular to the first assignment was my ability to show students examples of former student work. Since the Grade 11 group had produced such a wide, quality range of styles and media, I was able to give the Grade 9 students many different types of ideas on how they might choose to incorporate the golden section into their own project.

The second assignment, entitled *Linear Relations: Renaissance Perspective Drawing* (see Appendix F), was introduced with both a video entitled *Masters of Illusion* (Harper, 1991), which described three-dimensional Renaissance art techniques, and a software package entitled *Leonardo da Vinci* (Corbis, 1998), which entered the mind, works, and sketchbooks of this famous polymath. Students were taught how to draw using linear 1-, 2-, and 3-point perspective techniques. They were each given a large sheet of oversized grid paper on which to complete their second SMARTWORK, which juxtaposed linear perspective with linear equations on the Cartesian Plane. I created one sample myself in order to at least show a variety of ideas and media that could be used on the grid paper.

The third and final assignment, entitled *Geometric Patterning: Escher Tessellations* (see Appendix G), was introduced with selected segments from the two videos, *The Fantastic World of M. C. Escher* (Atlas Video, 1994) and *The Life and Works of M. C. Escher* (Acorn Media, 1999). Students were taught how to create tessellations using a variety of techniques such as constructing a template, using tracing paper with a grid, and using *Geometer's Sketchpad* in the computer lab. The final SMARTWORK was to be produced in any media of their choice using any of the above-mentioned techniques.

The three assignments featured seven different types of activities. Organized under the four Achievement Chart categories of *Knowledge/Understanding*, *Thinking/Inquiry/Problem Solving*, *Application*, and *Communication*, each assignment contained the following sections: definitions, related problems, home research, application questions or worksheets, a SMARTWORK creation, and both a written and verbal communication of the final product.

Each assignment was graded out of 50 possible marks, which were clearly delineated within each assignment. However, in order to facilitate a more meaningful, holistic, and comprehensive assessment for the benefit of students, parents/guardians, and the teacher, a final rubric was developed (see Appendix H). This rubric was based on the provincial *Exemplars* model, an initiative with which I was concurrently involved, and was used to summarize student achievement on all three assignments.

Video-Taped Presentations

On each of the three presentation days (October 30, December 18, January 22), a schedule was randomly predetermined for the order of presentations. All students who had submitted permission forms that included video permission always presented first so that the cameras could be turned off at the appropriate times. I used three different types of video cameras (regular VHS, hand-held VHS, and a digital camera) to ensure a quality recording. This was a fortunate decision as several technical problems did arise with two of the cameras during filming. For the sake of variety, each presentation day also featured a rearrangement of furniture (e.g. desks in semi-circles, as wheel spokes, and grouped in clusters) and a change in the focal point that moved to three different areas within the classroom (bulletin, side, and front boards).

Student Questionnaires

An open-ended questionnaire (see Appendix I), similar to the one distributed to the Grade 11 group, was given to the participating Grade 9 students following the completion, presentation,

and submission of the first assignment. This research instrument was completed by students during class, within a time period of approximately 15-20 minutes. The same questionnaire was again given to participating students following the second assignment. However, to allow more time and to encourage more reflective and meaningful comments, this questionnaire was sent home to complete. The mistake in this logic, in light of the age group involved (i.e., unlike the Grade 11 group), was soon realized as a much smaller percentage of the forms were ever completed and returned. Since those that were returned still were not providing me with the type of specific and valuable feedback for which I was hoping, I began to draft another instrument for the final assignment. Comments from the first two questionnaires were word-processed by question to facilitate easy comparison during analysis.

Final Student Survey/Questionnaire

The final student survey/questionnaire was both quantitative and qualitative in nature (see Appendix J). It featured the following questions: (i) one regarding past Grade 8 experiences, (ii) one designed to compare the three assignments in various ways, (iii) one which had students rank the different features of the assignments, and (iv) a series of questions that asked participants to comment on the different qualities of the integrated learning process. The final page once again asked three open-ended questions regarding likes, dislikes and recommendations for change. All 47 participating students completed this survey/questionnaire and the results, once tabulated and word-processed by question, did provide more meaningful, detailed, and easily accessible information for the research study.

Interviews

From January 8-19, 2001 semi-structured interviews (see Appendix K) were conducted with 12 of the 18 Grade 11 students who had experienced the pilot study in the spring of 2000.

Following these interviews, and using a very similar set of questions (see Appendix L), nine individual and five group interviews were conducted with Grade 9 students from January 23-25.

A stratified random sampling technique was used to determine the selection of students. Participating Grade 9 students were first sorted according to midterm achievement on the report card. From this ranking, two students from each of the five mark categories (i.e. 50-59%, 60-69%, 70-79%, 80-89%, 90-100%) were randomly selected to be interviewed individually. Following these interviews, clusters of students from both classes were randomly assigned to six scheduled group interviews. This was done to both accommodate as many people as I could possibly see during that final week of classes, and to provide a somewhat different forum for the interview process. Although each group originally consisted of four students, sometimes one or more of these students was not present for the interview, and on one occasion, an entire group, scheduled for a morning interview, missed their appointment.

All 28 interviews were audio-taped and later transcribed. These comments were also word-processed, according to the questions, to facilitate easy comparison during analysis.

Response Form and Open House for Parents/Guardians

The idea of an Open House for students of mathematics, similar perhaps to a visual arts exhibition, a music recital, or a science fair, had always intrigued me. Without this type of exciting, deeply anticipated, social, and culminating activity – so habitual for these other disciplines – mathematics has always been at a disadvantage regarding student motivation. And so, with this educational disparity on my mind, and with an earnest desire to facilitate a response from parents/guardians, I decided to follow through on the idea of an Open House.

A letter regarding the Open House (see Appendix M) was sent home to parents/guardians immediately following the Christmas break. The event was scheduled for the evening of January 23rd, 2001 from 7-9:00 p.m. A small response form was added to this letter and five of these were

returned with written comments. Several other parents/guardians indicated, via their children, that they wished to speak with me in person during the Open House event. Invitations were also made to the local school board superintendent, the principal of the secondary school, the department head of Mathematics, and my thesis advisor.

Schedules and Procedures

As previously described, the three integrated assignments were completed by students from two Grade 9 academic mathematics classes within the first semester of a regular secondary school year. Student SMARTWORK presentations to the class were video-taped, and student projects were then put on exhibition around the classroom. Identical questionnaires were given to participating students after each of the first two assignments. After the third assignment was finished, a final survey/questionnaire was administered.

Towards the end of the semester, semi-structured interviews were conducted with a stratified random sample of students. Although similar to the questionnaires, the interviews allowed for a different form of response, increasing the validity of the research. Finally, students and parents/guardians were invited to attend a mathematics Open House that showcased the student work, contextualized the research, and facilitated discussion regarding parental/guardian impressions of the assignments and the integrated learning process.

Validity and Reliability Check

Mills defined validity as “a test of whether the data we collect accurately gauge what we are trying to measure” (2000, p. 90). He further defined reliability as “a measure of the consistency with which our data measure what we are attempting to measure over time” (p. 90). Although Mills described the various Action Research interpretations of validity held by Guba, Maxwell, Anderson, and Wolcott, it is the latter’s strategies for ensuring the validity of Action Research that I found most helpful. Wolcott’s eight strategies for research validity are the

following: (i) talk little, listen a lot, (ii) record observations accurately, (iii) begin writing early, (iv) let readers *see* for themselves, (v) report fully, (vi) be candid, (vii) seek feedback, and (viii) write accurately (Cited in Mills, 2000, pp. 81-82).

This research study involved data collection from seven different instruments. The field notes, integrated assignments, video-taped presentations, questionnaires, interviews, and parent/guardian response forms provided me with a rich source of primary, inter-related, qualitative data. Themes regarding student perceptions and qualities of the integrated learning emerged from this data web during analysis.

The final student survey added a quantitative agent to this primarily qualitative case study. It provided me with actual statistics regarding student perceptions, which were cross-referenced with observations from the analysis of the aforementioned instruments. The synthesis of information from these seven sources increased the validity of the data and led to more reliable conclusions from the research on students' perceptions of integrated learning.

Data Analysis and Reporting Procedures

In a qualitative case study, analysis of the data is often one of the most overwhelming yet meaningful stages of the research. As Mills (2000) noted,

> This task is often daunting for action researchers who, engaged in the regular, ongoing collection of data, must change their focus and adopt a more analytical and interpretive lens. They must move beyond the description of the phenomenon they have studied and to make sense of what they have learned. (p. 97)

He further highlighted some key techniques for teachers doing data analysis within an Action Research framework: (i) identifying themes, (ii) coding surveys, interviews, and questionnaires, (iii) asking key questions, (iv) doing an organizational review, (v) concept mapping, (vi) analyzing antecedents and consequences, (vii) displaying findings, and (viii) stating what is missing (pp. 100-103).

Of these eight strategies, the only one that was used to a lesser extent in this study was the organizational review. Apart from understanding the policies and procedures of the secondary school and the mathematics department, something with which any teacher would be familiar, the only organizational review that occurred was with regard to the Ministry's curricular documents.

Themes were identified with regard to the unique qualities of the integrated process, curriculum writing, and the elements of personal professional development, all of which appear in the latter sections of the chapter 5 analysis. Although the questionnaires, interviews, and the survey were structured in such a way as to separate issues and to provide clear questions regarding student perceptions, coding of the responses to the aforementioned instruments did take place to help recognize patterns within the data.

The research questions also underwent review and revision. The original intent of the study focused more on student achievement as a result of integrated learning. Accordingly then, the supplementary questions regarding creative mathematics application and mathematics communication were worded in such a way as to further analyze the effects on student achievement. As the study progressed, however, it became obvious that the bulk of the data more accurately dealt with student perceptions of integrated learning, as opposed to effects on student achievement.

The key questions in this research were then reworded, as were the questions featured on the questionnaire and in the interview (e.g. the question "Did you feel that the assignment helped you to creatively apply your mathematics learning in the classroom?" became, "In your opinion, did the creative application of the new learning help you to better understand the mathematics concepts involved?"). These changes were met with the approval of the Ethics Committee and were carefully considered in consultation with my thesis advisor.

As a visual learner, concept mapping is a preferred strategy that I use whenever possible. Therefore, it was only natural for me occasionally to illustrate the various aspects of the research, as evidenced in the sketches found in the field notes. For example, as the qualities unique to integrated learning and curriculum writing came more clearly into focus, a concept map of interconnected ideas and reworked titles was given much attention in the field notebook, in order to help clarify my thoughts and ideas.

Antecedents were thoroughly analyzed in the literature review, theoretical framework, and related studies sections of chapters 1 and 2. Missing elements in the research were addressed both in an ongoing fashion, as in the restructuring of the research instruments, and in a final sense, when discussing limitations of the study and future possibilities.

The display of findings has taken several forms in this research study. The quantitative findings of the final student survey were interpreted using statistical analysis. Measures of central tendency, frequency ratings, and the tally scores for the Likert scale responses all provided meaningful data during the analysis stages.

The Open House served as a very visual and public display of the assignments and student work. In a more formal sense, this written thesis, and related papers that may potentially follow will afford me a chance to share the findings with colleagues and with those students and parents/guardians who may be interested.

Chapter Four: Presentation of the Findings

Overview of the Data

As mentioned previously, the variety of research instruments used in this study provided triangulation that supported the validity of the research analysis. Table 1 shows a breakdown of the data collection from the seven instruments. In total there were 256 pieces of data collected, or 294 if one includes those from the pilot study.

Grade 11 Pilot Study		**Grade 9 Research Study**	
Assignments	18	Assignments	137
Questionnaires	8	Questionnaires	60
Interviews	12	Interviews	26
		Final Surveys	47
		Parent/Guardian Response Forms	5
		Field Notebooks	1
		Video Footage of Presentations	1.5 hours

Table 1. Overview of the seven research instruments by number.

Presentation of the Findings from the Assignments

Each of the three integrated assignments was created by combining ideas from an historical time period, a character(s) from that period, a visual arts component, and a mathematical skill, or skill set, that was to be reinforced in the learning.

Each of the three integrated assignments (see Appendices E, F, and G) was directly tied to the *Ontario Curriculum Grades 9 and 10: Mathematics* (1999) document. Relevant expectations,

both overall and specific, were listed on the first page of each, along with a rationale that presented excerpts from a section entitled, *The Place of Mathematics in the Curriculum* (pp. 3,4), of the above-mentioned document. These excerpts highlighted the importance of investigating interdisciplinary connections, and of emphasizing the communication of mathematics in the secondary classroom. Several key publications used to construct the assignments were also listed as resources on this opening page.

The second page featured a copy of the Achievement Chart, with which the remainder of each assignment was organized by category. The *Mathematics Knowledge/Understanding* sections asked students to define 10 terms or names that were related to the art, history, and mathematics involved in each assignment. The *Mathematics Thinking/Inquiry/Problem Solving* sections required students both to solve interesting related problems and to research relevant mathematical phenomena outside the classroom. The *Mathematics Application of Concepts* sections allowed students to practice the integrated art/math skills needed to complete the final SMARTWORK. And finally, the *Mathematics Communication* sections required students to communicate their final projects, both in written form and in a brief presentation to their peers.

The first two sections were marked out of 10 and the last two sections, being the foremost foci of the research, out of 15. Each assignment received a final mark out of a possible 50, and at the conclusion of the research study each student received a summative assessment rubric (see Appendix H) which reported their overall level achievement for each of the seven criteria.

Assignment # 1: Ratio, Proportion and the Golden Section

There are four strands within the new Grade 9 curriculum. The first assignment sought to reinforce expectations from the *Number Sense and Algebra* strand, particularly those dealing with ratio and proportion. This assignment was centered around the character of Pythagoras, the historical period of ancient Greece, and the art/math concept of the golden section rectangle.

In the student survey (see Appendix J), this assignment was shown to be the only one of the three in which the art/math technique involved had not been taught to any of the 47 students in elementary school. Although ratio is introduced in the *Ontario Curriculum Grades 1-8: Mathematics* document (1997, p. 27) as part of the Grade 8 *Number Sense and Numeration* strand, no mention is made of the golden ratio/rectangle in either the mathematics or the visual arts Grades K-8 documents. Therefore, the lack of student exposure to this art/math phenomenon was to be expected.

Assignment # 1 was voted the assignment that: (i) required the most amount of effort, (ii) was most interesting with regard to seeing the works of peers, and (iii) best reinforced the new mathematics learning. The survey also showed that students perceived that the first assignment was the one in which they best communicated, were most creative, and were most thorough. It was voted as neither the most favourite assignment nor the least favourite assignment, leaving it in the central position of the three regarding student preference.

This assignment had an overall mean grade of 41/50, a median grade of 43/50, and was therefore the assignment on which students performed the best regarding marks.

Assignment # 2: Linear Relations and Linear Perspective

The second assignment sought to reinforce expectations from the *Analytic Geometry* strand, particularly those dealing with slope, equations, and the Cartesian plane. This assignment was centered around the two characters of Leonardo da Vinci and René Descartes, the historical period of the High Renaissance, the art technique of linear perspective drawing, and the mathematical skills of calculating slope and determining the equations of lines.

In the survey, 23 students indicated that they had been instructed in linear perspective at the elementary level. Since this visual arts technique is introduced at Grades 6 and 7 in the *Ontario Curriculum Grades 1-8: The Arts* document (1998, pp. 40,42), it was somewhat

disappointing, yet not surprising, that over half of the student population had not received instruction in this artistic science.

When asked which assignment was presented by the teacher in the most effective and interesting way, students voted Assignment # 2 as last. It was also voted as least favourite among students. On the other hand, it finished very close, by number of votes, to being the assignment in which students felt they were most creative, and also in which they felt they had best communicated.

This assignment had an overall mean grade of 33/50, a median grade of 38/50, and was therefore the assignment on which students performed the poorest regarding marks.

Assignment # 3: Geometric Patterning and Escher Tessellations

The third and final assignment sought to reinforce expectations from the *Measurement and Geometry* strand, particularly those dealing with interior/exterior angles and geometric relationships. This assignment was centered around the character of M.C. Escher, the historical period of twentieth-century Europe, and the art/math concept of tessellations, using transformational geometry.

The survey showed that approximately one third of the students had been introduced to tessellation constructions in elementary school. Transformational geometry is first mentioned as part of the Ontario mathematics curriculum at the Grade 1 level (1997, p. 43), where general terms are used, and then begins at the Grade 2 level where a more specific vocabulary (e.g. reflections, rotations, translations) is adopted (1997, p. 44). Tiling patterns, known as tessellations, are to be formally introduced as part of the Grade 7 *Geometry and Spatial Sense* strand expectations (1997, p. 50). Since introductory lessons for golden rectangle construction, linear perspective drawing, and tessellation tiling were given to all students as part of my

integrated assignments, prior elementary school experience in these skills was not a necessity, but merely functioned as a bonus.

The third assignment received the lowest number of votes for the following categories: (i) requiring most effort, (ii) most interesting to see others' work, (iii) best reinforcing new mathematics learning, (iv) best communication of mathematics concepts, and (v) most thoroughly completed. However, it was voted as the most favourite of the three assignments. It was nearly tied in votes with the first assignment for being the one in which students felt they had been most creative, and it was voted first as the assignment that was presented by the teacher in the most effective and interesting way.

This assignment had an overall mean grade of 37/50, a median grade of 40/50, and was therefore the middle assignment regarding student mark achievement.

Assessment by Achievement Chart Category

The marks given for the assignments were recorded separately by Achievement Chart category. By totaling the marks of all students within each of the four categories and then by comparing these totals as adjusted percentages, the results showed that overall student achievement in the three assignments was found to be in the following order, from strongest to weakest performance:

1. Mathematics Application of Concepts	79%
2. Mathematics Knowledge & Understanding	75%
3. Mathematics Communication	74%
4. Mathematics Thinking, Inquiry, & Problem Solving	70%

These results, of course, are based on the way in which I defined the four Achievement Chart categories in the actual creation of the assignments, and on the way in which I assessed the sections in each category.

Assessment by Rubric Criteria

The final assessment rubric, developed just prior to its use, allowed me to compare student level achievement across the seven different criteria that were based on clusters of the ten selected expectations (similar to rubrics developed for the provincial *Exemplars* performance assessments). Table 2 contrasts the student rankings of the seven activities (criteria) from the survey, with the actual level achievement ranking of the criteria as evidenced in the results from the rubric assessment.

Overall Student Rankings of Criteria (Most Favourite to Least Favourite)	**Overall Student Level Achievement Ranking (Highest Level Achievement to Lowest)**
1. SMARTWORKS	1. SMARTWORKS
2. Definitions	2. Verbal Communication
3. Application Practice (Worksheets)	3. Definitions
4. Problem Solving (Thinking/Inquiry)	4. Written Communication
5. Verbal Communication	5. Home Research and Calculations
6. Home Research and Calculations	6. Application Practice (Worksheets)
7. Written Communication	7. Problem Solving (Thinking/Inquiry)

Table 2. Student ranking of criteria compared with student level achievement regarding criteria.

Presentation of the Findings from the Video-Taped Presentations

The presentations of the three integrated assignments were generally brief, lasting anywhere from 20 seconds to several minutes. As the participant-observer, an effort was made on my part to extend each presentation by asking each student a few relevant questions regarding their project, or by adding a related comment.

In the first set of presentations, the students seemed both anxious and excited. They were unfamiliar with such an activity within the framework of a mathematics class and were not sure what to expect. I asked students to stand on a small platform so that the cameras could more clearly tape their SMARTWORKS while they presented. This likely added to the trepidation for those students already dreading the idea of public speaking. In the two subsequent sets of presentations, I abandoned the idea of the raised platform, and simply rearranged the desks in such a way as to facilitate clear lines of sight for the cameras.

Some students had fully scripted their verbal presentations and simply read these to the class, with differing degrees of rhetorical success, depending on their presentation skills. Others had obviously done little or no preparation for the event, and had therefore very little to say.

Because the second assignment was more prescriptive, the final projects were more similar in content and appearance. This had a somewhat negative effect on this set of presentations, since each student made very few original comments, producing an overall effect of dry repetition. The third and final set of presentations featured projects that were much more visually stimulating and varied, leading to more animated comments and questions. Students described the transformational geometry used in the template constructions for their tessellations.

Besides filming the three sets of presentations, I also was able to film some of the cooperative group work sessions, the classroom exhibitions of student work, and the setting for the Open House evening. Student perceptions regarding the presentations and the communication of mathematics were quite varied and these will be dealt with at length in chapter 5.

Presentation of the Findings from the Student Questionnaires

Grade 11 Pilot Study Questionnaires

Findings from the pilot study questionnaires submitted optionally by eight of the 18 students in the spring of 2000, contained information that was very helpful during the writing of the integrated Grade 9 curriculum several months later.

Although a few Grade 11 students initially felt somewhat overwhelmed by the novelty and expectations of the math/art project, four positive themes emerged from the questionnaires regarding their enjoyment of the golden section assignment: (i) the integration of mathematics with other subjects, (ii) the variety it introduced into the course repetition, (iii) the creativity required and encouraged, and (iv) the classroom presentations of the assignments.

The following are a few quotes from the questionnaires that relate to the list of above-mentioned factors.

“The project helped to liven things up a bit. It was great not to have to do something structured for once and to be able to think creatively, while still doing math.”

“It made me realize that mathematics has so many applications outside of just math.”

“I learned that math isn’t always a big scary monster that lurks in dark corners. Math can be very beautiful and exciting.”

“I think this project was a good idea. I agree with integrating other subjects, such as art, with math. I think by doing so, math is more fun, and not so repetitive.”

“I think that it’s good to try to express math in some other way. It tells you how well you understand it.”

Regarding the display of samples at the beginning of the assignment, the students shared a variety of opinions. Some liked the absence of examples while others thought that a wide variety would be beneficial to all students.

“I totally feel that more examples would hamper creativity. Leaving it wide open allowed us to think for ourselves.”

“I disagree. Lots of examples gets some ideas going for those who have no idea how or where to begin. The samples this year might get next year’s class really rolling. It’s such a wide open field that it’s difficult to narrow things down to get an idea.”

These questionnaires also produced the following six recommendations, all of which were addressed in the creation of the Grade 9 assignments:

- More detailed instruction regarding the construction of the golden section rectangle
- Clearly defined assessment – how the marks would be allocated
- More time to complete the assignment
- More exciting and interesting initial presentation of the assignment
- Direct connections to the course outline and government curriculum
- Interviews (student/teacher conferences) following the presentations

This cycle of action, evaluation, and reaction is in keeping with the Action Research philosophy as discussed earlier in chapter 2. In *Action Research: School Improvement through Research-Based Professionalism* (Delong & Wideman, 1998), Morgan summarized this positive symbiosis of teaching and research in the following way:

> Action Research allows the teacher as a professional to become an integral component in the process of change. . . . You are indeed empowered. Action Research provides an educational theory that encourages teachers to modify their teaching practice and, as a direct result, improve student learning. (p. 58)

The Grade 9 assignments that followed the Grade 11 pilot study would in turn bring new insights, leading to further changes and improvements in curriculum and pedagogy.

Grade 9 Questionnaires

The Grade 9 questionnaires revealed a much wider spectrum of opinion. Comments made by the 47 students were often less serious and less sophisticated than the Grade 11 responses; a reality to be expected from students at this level of age and secondary school experience.

Similar themes regarding the enjoyment of creativity, variety and thematic integration were also evident. For example one student commented about integration, “I liked the way math and art was tied together. I also like hearing about math history, and how math ties in with almost everything.” Another expressed her thoughts on application, “I liked being able to create something using interesting mathematics. I liked seeing what other people can do with mathematics too.” However, the presentations of the assignments were less favoured by many of the Grade 9 participants, at least at the conclusion of the first assignment.

Beyond these comparisons, several other significant patterns emerged from the Grade 9 questionnaires. The understanding of mathematics concepts that students perceived to have occurred as a result of the integrated assignments, was highlighted in several distinct ways. Students commented on the personal benefits of the four following types of learning as experienced to a greater or lesser degree in the three assignments: visual learning, tactile learning, oral learning, and social learning. The following four comments make reference to these four types of learning respectively:

“Yes, I think the learning was reinforced, because it seemed easier with pictures to learn.”

“I think it helped because not only did I write it down and learn it, but I got to do it hands-on too.”

“I learned a lot about the golden ratio and how to find it. I also learned how to communicate my mathematical thoughts into visual arts.”

“I found that with people showing math from their point of view the class was in a more positive atmosphere.”

The students who did not like the assignments made comments regarding such things as a lack of connection with the “real” curriculum, lack of sufficient time to complete the project, lack of sufficient direct instruction, and insufficient skills in art that affected their ability to be successful. One student commented, “I thought that it was a waste of my time. What is it useful for? What do we learn that we will use later in life?” Whereas most of the negative comments were made by two particular students, who had not demonstrated very high levels of energy on any classroom assignment up to that time, there were a few exceptions to this rule. One student, who received some of the highest marks on the assignments, still maintained that because the projects demanded so much time and effort she could not support the idea of this type of integrated approach to learning. Another disgruntled, but very hard-working student, claimed, “It distracted me because art is really hard and math isn’t easy either.”

However, as supported by the final survey and the interviews, a large majority of Grade 9 students enjoyed the assignments for the various reasons detailed above. One determined student adequately summarized the challenging nature of the assignments, “I learned that it sure takes awhile to get a perfect golden rectangle.” A more optimistic friend concluded, “I learned that some mathematics stuff is really cool.”

Presentation of the Findings from the Final Student Survey

Much of the findings from the first page of the student survey (see Appendix J) have been already presented in an earlier section in conjunction with the presentation of the findings from the assignments. The second page of the survey featured 10 questions in a Likert scale format. Triangulation with the questionnaires and interviews, permitted me to confirm the responses on this section of the survey.

Figure 2 displays a graphic representation of the results of the four survey questions that related most directly to the research questions as presented in chapter 1. These four survey questions were as follows:

- Q4: The three integrated assignments helped me to creatively apply the new mathematics learning in each.
- Q5: The three integrated assignments helped me to better communicate (written & verbally) the new mathematics learning in each.
- Q6: The three integrated assignments increased my motivation for learning in this course.
- Q13: This integrated approach to mathematics education, using visual arts, should be used regularly.

If the selected responses of *strongly agree* and *agree* are combined, these four questions received a “positive” response of 85%, 68%, 64%, and 83% respectively.

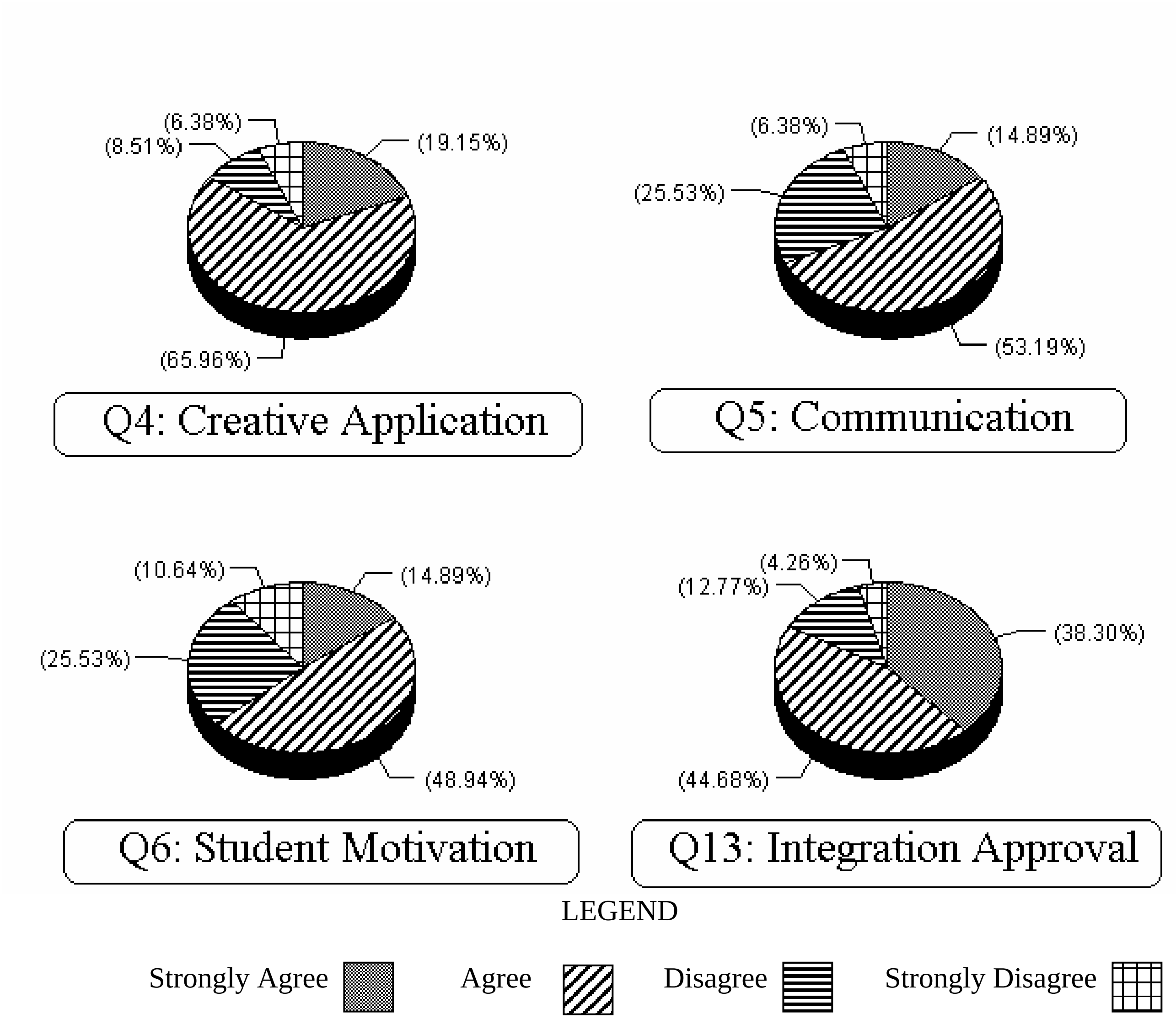

Figure 2. Results from the final student survey.

The remaining survey questions revealed that the majority of students: (i) found the group work sessions to be very helpful (94%); (ii) thought the exhibition of student work in the classroom was an important part of the assignments regarding motivation (91%); (iii) disapproved of a required Internet research component for every assignment (64%); (iv) disapproved of a required *Geometer's Sketchpad* component for every assignment (55%); (v) did

not consider the integrated assignments to be a negative distraction from the mathematics learning (87%); and (vi) did not prefer assignments void of art, history, and integration (94%).

Presentation of the Findings from the Student Interviews

The student interviews provided me with a valuable cross-reference to the student questionnaires, and served to increase the reliability of the research with regard to student perceptions of an integrated curriculum. For the sake of comparison and continuity, I would like to document five different character studies, using both the questionnaire and interview comments made by these selected individuals. These character studies will highlight what I consider to be representative samples of five types of mathematics students: an average student (Level 3), an exceptional student (Level 4), a weak student (Level 1), a repeating student, and an adult student.

In the *Ontario Curriculum* (see Achievement Chart, *The Ontario Curriculum Grades 9 and 10: Mathematics*, 1999, p. 39) Level 3 has been defined as the provincial standard; Level 4 as beyond the standard, or superior; and Levels 1 and 2 as below standard, but still passing. The names of students in the following character studies have been changed to ensure anonymity.

Tetra

Tetra is somewhat of an enigma. Having enjoyed much success in elementary school mathematics, she was immediately perturbed by the nontraditional, integrated approach and expectations. A parent conference interview confirmed the facts that Tetra used to love mathematics in Grade 8 when there was simply straightforward textbook questions and that she really felt overwhelmed by all of the art, history and communication components in the new assignments.

Although she claimed to dislike the integrated aspects, the written explanations, and the verbal presentations, Tetra produced some of the most excellent work in the class on all sections

of the assignments. Her SMARTWORKS were creative and thorough (especially her three-dimensional golden rectangle constructed with Rice Krispies), her written communication was word-processed and detailed, and her oral presentations were carefully scripted and well-delivered. Notwithstanding her Level 4 achievement across the board on the assessment rubric for the three assignments, Tetra indicated on the survey that she did not feel that they helped her to better communicate, create, or be more motivated. Overall, she disagreed that this integrated approach to mathematics education should be used regularly and claimed that she considered it a negative distraction from the course.

The following comments made during the interview, or from the questionnaires, perhaps help us to begin to understand Tetra and her perceptions of the assignments:

"The assignments were OK, but the first two I think took a lot of time to do – but you learned a lot. The presenting part I don't like. There was a lot of art and history in them too which I found really hard to do because I'm not good at art or history."

"It's hard for me to write things down. I know what I did but I don't know how to write down what I did. . . . I find it hard to express myself with mathematical words."

"Compared to other years in math, where it was just writing and doing things – you don't actually get to do assignments like this – so I kind of liked that we had to do the assignments. I don't really know how to explain it – I just don't like the presenting part."

"I liked that you showed other people – it wasn't that we did it and it was over.'

"I liked that you didn't have to do them a certain way – you got to do it how you wanted."

I am of the opinion that Tetra, a very talented but taciturn student, was originally disturbed by both the unfamiliarity of the mathematics projects and by the dreaded oral presentation component. As the semester progressed and she became more comfortable with the idea of multi-faceted assignments, at which she was successful, Tetra relaxed just enough to let

herself enjoy the aspects that catered to her interests (e.g. group work, exhibition of student work, and the creativity involved). She finished the course with 79%, a slightly above-average student with all things considered.

She represents, for me, that typical Level 3 student who works hard to succeed in mathematics, struggles with unfamiliar contexts and expectations (especially verbal and written mathematics communication), yet greatly enjoys – sometimes secretly – certain aspects of the integrated curricular approach. For many of these types of students, not only do the assignments increase their potential for academic success in the course, but they also provide the classroom variety, coupled with creative expression, that makes the projects meaningful and memorable. This is likely why many of them voted in favour of the integrated curricular approach in the final survey and also explains their many positive comments. However, that being said, Tetra’s experience and remarks still remain rife with contradictions.

Octavian

Although not formally identified as gifted, Octavian is an exceptional Level 4 mathematics student. He scored perfect on all three assignments and finished the course with one of the highest marks of 91%. Apart from his nervousness throughout the verbal presentations, Octavian expressed positive remarks regarding almost all aspects of the assignments, in both the interview and on the questionnaires. He felt that the creative application of mathematics and the communication of mathematics, as experienced in the assignments, both helped to reinforce the new learning. The following are a few of his comments:

“I thought they were a good idea – to learn about different subjects and to get students interested in different categories – just a different way of learning instead of notebook work all the time.’

"I think it actually helped to reinforce the learning. It makes you look over it more and understand what you did."

"I think the writing part helped reinforce the learning. I actually understood it better after I wrote it, because I had to put it into words."

"I did not like to present the assignments, but it is good to speak in public."

Octavian's only other recommendations for the assignments were a request for more examples shown to generate ideas, and a desire for "maybe more equations to perform while doing the projects (i.e., more problem-solving)." His first assignment chronicled the major events of the last century as they spiraled out from the sinking *Titanic* at the center of the golden rectangle to pictures of the new digital technologies found in his largest square. In his second assignment, not only did he complete the lengthy list of prescriptive instructions, he also added several new inventions including a *hoverboard* with a water-powered engine. In his own words, "The other inventions are also neat because I think that they could actually be made if someone put their mind to it, even if they don't have any practical uses. What a great idea to just make some crazy inventions, and have fun while you're inventing." In his third and final assignment, Octavian created two tessellating patterns using unique forms of transformational geometry that no one else had attempted.

This integrated approach to education allows students like Octavian the maneuverability to push beyond the baseline expectations to explore the limitless terrain of possible ideas, while still learning the various expectations of the required curriculum.

Cubert

Notwithstanding his final course mark of 52%, one could honestly say that Cubert's high achievement in the integrated assignments (43, 45, and 43 out of 50 marks) was merely a reflection of his dedication and tireless effort. He was not a strong mathematics student by any

means, perhaps even misplaced by level, yet there must have been some strong connection that occurred between him and the learning, which led to his impressive performances on all three assignments.

His parents were very supportive and had worked with him and his ideas, but it was Cubert himself who stayed late to discuss possibilities with me, who came back often after school, and who was the first person to explore the related websites and computer software. I was especially proud of this student when, after he had totally missed the point on his first attempt at the tessellation project, and after he had suffered from the related laughter of fellow students during his presentation, he agreed to revisit and resubmit the piece according to a few of my suggestions. In my opinion, at least, that required real courage and profound commitment on his part.

Cubert was another student who absolutely feared the public speaking portions of the assignments. His discomfort in sharing verbally, even in the private interview process, was evident. Nonetheless, the following are a few of his brief but informative comments:

"I don't really know if the communication helped my learning – I got nervous up there."

"I liked the course more because of the assignments – it was more interesting."

"It increased my motivation."

"I liked measuring all the lines and drawing a lot. I also liked the computer stuff."

Several other students with final course grades below 65% did very well on the three integrated assignments. Perhaps this is because the assessment of the assignments permitted the attainment of "easier" marks. Perhaps the ongoing assistance from parents, peers, and the instructor helped these students to achieve a level of success on the assignments that was impossible for them on regular class tests. Or could it be possible that this type of integrated learning provides a window through which struggling, Level 1 students can catch a glimpse of

the beauty and exciting nature of mathematics, leading to increased personal motivation, effort, and success?

Dodeca

Whereas the first three students were selected from the Grade 9 participants, the last two will be chosen as representative characters from the Grade 11 group.

Dodeca had already failed this particular mathematics course once. A talented visual arts and drama student, she perceived herself as a mathematics failure, and was only retaking the course because of college requisites and parental encouragement. Like so many other secondary students that I teach from year to year, Dodeca severely disliked mathematics and merely wanted to survive this second wave of Grade 11 with a passing grade. In her own words, more concisely,

"Personally, I'm not a very big fan of math. So since I really like art, it kind of catered to my interests a little bit – made my ambition kind of move up a little bit more than normal. I started looking forward to coming to class when we got to work on it – started looking forward to working on it myself."

"If it was distracting, it was beneficial to get away from all the hard edges that everyone perceives math to have – but it doesn't always – that's what that class showed me. Before then . . . my total idea of math was really low, so in having this project – if it was a distraction, it was a great distraction."

Dodeca passed the course with 57%. Apart from the "dryness and length of the Griffiths article," which was required reading as a springboard for the Grade 11 assignment, she had no complaints or recommendations about the entire experience. This notion of a positive distraction surfaced in a large number of the interviews, in both the Grade 9 and 11 classes. While the integrated assignments were perceived as sometimes causing a decrease in premium time available for regular mathematics homework, the trade off in terms of variety and creativity was

often deemed acceptable. For a repeating and disinterested mathematics student like Dodeca, the integrated initiative may not have directly helped her pass the course, but the metaphoric light in the mathematics tunnel was indubitably for her a bright spot in a dark place.

Icosa

Icosa was an adult student who had returned to secondary school to upgrade marks in light of her desire to become a medical doctor. She was simultaneously completing university courses by correspondence in subjects such as chemistry, biology, and physics, as well as raising a child as a single mother and working a part-time job. Her careful work habits, dedication, and time-management skills were extraordinary, and certainly all contributed to her 87% final average in the course. During the interview, she shared with me that although she felt she was always a very capable student, especially in the academic disciplines, she had not been intrinsically motivated during her first encounter with secondary school. Now that she had a focus and a personal dream to pursue, she was determined to give everything her utmost effort. This included, of course, the small, integrated assignment that became part of the Grade 11 course in the spring of 2000.

I highly value the opinion of Icosa regarding the assignment because she represents to me someone who has, educationally speaking, been there and back again: someone who can therefore bring a wealth of experience to bear on anything upon which she decides to comment. Some of her insights read as follows,

“In our particular course, as a first effort [researcher’s first integrated effort], it was a little on the disjointed side at times, and that’s entirely to be expected. You’re feeling out the area, you’re trying to gauge the time allowed to get the tie-ins together. But definitely, I could see where it was going. Math and art are definitely intertwined.”

For obvious reasons, Icosa felt that more time would have been helpful to complete the assignment. I appreciated her honesty regarding both the effectiveness of the presentations and the exhibition of student work in the following quotations:

"I don't know that the presentation helped [reinforce mathematics learning] as much – hard to say. I don't think some members of the class understood what was being asked quite as well as some of the other ones. I guess you're going to find that – there's going to be a few that can twig into the artistic aspect of it and grasp the connection between the mathematics and the art in their work; and then there were a few others who were kind of out in left field."

"I think there again [exhibiting student work] you're going to get a mixed bag because the few people who didn't understand the concept or who gave it a slapdash effort would probably have felt a little down about having their art work up on the board. But you definitely had something there, because again you could see the ideas that other people had – you could see a particular angle or viewpoint that they had on something that you may not have thought of yourself."

Finally, Icosa reflects on her project in which she creatively combined the ideas of modern String Theory, the violin with sheet music and text, and the golden section rectangle as a design blueprint:

"I liked the freedom to explore. I liked the idea that you could take a concept . . . and you could actually develop a viable art project out of it. I liked the idea of putting the theory into a practical form."

In her unique position of having experienced secondary school mathematics years before, as well as concurrently participating in post-secondary studies, Icosa stressed her approval of the integrated approach. She recognized and verbalized its potential as an effective educational tool

that could facilitate regular praxis – theory into practice – at the classroom level. Her enthusiasm, amidst the pressing realities of her life, was a very real encouragement to me.

Presentation of Findings from the Open House and Response Form

The Open House occurred January 23rd, 2001. Approximately 60 people were in attendance that evening, with 18 student families represented. Of these 18 families, ten were connected with female students and eight with male students. Also in attendance were a retired mathematics teacher (on his birthday), my family, two other teachers and a janitor.

That night, mathematics room 202 featured contextual explanations of the research and each assignment, sample student work, background books and materials, and the three exhibited series of SMARTWORKS. Refreshments were available which included coffee, juice, and homemade golden rectangle shortbread cookies. While quiet classical music played in the background, and during those moments in between conferences, I had the opportunity to simply watch students and parents discuss, admire, and snack on mathematics. The fortuitous afterthought of an Open House was definitely a personal highlight of the research for me.

Although only five of the parental response forms, located at the bottom of the information letter, were returned to me with comments, I did have a chance to speak with many individuals during the Open House itself. One student attended the evening with her parents, two grandparents (separate sides of the family), and her older brother who had experienced the Grade 11 assignment several months earlier. In talking to these parents, I learned that their daughter, who traditionally struggled with mathematics, had really enjoyed the course and that they wanted to see what she was so excited about.

Another family of four attended the Open House with their son acting as private tour guide. The mother later wrote the following, “The math assignments were very interesting. I enjoyed watching the different angles, techniques, and abstract that went along with each project.

The funniest part was watching my son try to write his name backwards." Funnier still, in my opinion, was the son's own comment during a subsequent interview, "My dad hated it. He favours more 2 + 2 = 4 on paper and thinking about it in your head – not drawing. My mom liked it because she's a real big art critic." This student not only had to act as a guide, but apparently also as a mediator and keeper of the integrated peace.

Yet another couple, this time the parents of a student with one of the highest final averages in both classes, shared these sentiments,

"We found the SMARTWORKS to be extra work in combination with her other math homework and assignments. We did find this very challenging, but yet also interesting to find out that there is more to math than adding and subtracting. After seeing all of the displays and SMARTWORKS, we were quite impressed. Great work!"

The mother of a very weak mathematics student who liked and did well on the assignments, wrote the following note, "We found the assignments very interesting and enjoyed helping him with them. They were much more fun than regular math homework and we feel he learned a lot from them."

Two pressing concerns were evident throughout the various conversations with parents that I had during the evening. Like the department head months before, parents wanted to know if these assignments were curricular or extracurricular in nature. Furthermore, they were curious as to how these assignments would co-align with the provincial mathematics assessment that their children had just experienced. To the first concern I reiterated the section from the introductory Letter of Informed Consent which they had signed and which stated that the assignments were designed to reinforce stated expectations from the Grade 9 curriculum, and were therefore curricular in nature.

With regard to the second concern, I informed parents that much of the provincial assessment had featured questions that required the written communication of mathematics using numbers, symbols and words. To this end, I explained that the three integrated assignments had provided students with ongoing practice in the verbal and written communication of mathematics learning throughout the semester, which could only serve to increase their achievement on these sections of the provincial test.

The student interviews revealed some other aspects of the Open House, regarding the perceptions of students. The following quotations highlight these thoughts.

"Yes, I think it's [Open House] a good idea because some of the parents have no idea what we're doing. They can see the materials that you have and see how much effort you have put into it and how much effort we have put into it as students. They get to see the different things. I know my mom, she helps me sometimes and she'll get to see all the other things too, that other kids did."

"Yes, because we can tease our parents because we know more than they do."

"I think it was really important because my parents were really impressed with everything. At first they didn't know what I was doing, so I think they were happy. I think it was really good that you had that because everyone could come in and see what we were doing. I know some people knew we were doing extra stuff but they didn't really know what it was we were doing."

"Sometimes they [parents] didn't even see the SMARTWORKS before they were handed in, so it was nice for them to see them. I liked showing them all the stuff that I had done and everyone else's that I knew."

"I liked it. It gives you something to work for – to know that people are actually looking at them."

When the last visitor had left room 202 at about 9:30 p.m., and before the clean-up process began, I spent half an hour documenting the entire experience in the field notebook. The following quotation, taken from these pages, summarizes my thoughts that evening.

"More magical for me still, was watching students lead their parents around the room, explaining math ideas, art techniques, their colleagues' ideas, and their favourite pieces. I would have liked to quietly follow everyone around and simply to have listened to the conversations, but I found myself constantly talking to people for two hours instead."

Presentation of the Findings from the Field Notes

Findings from the field notebook will be presented by month from September 2000 to January 2001, highlighting key events, issues, concerns, anecdotes, and changes that transpired as the research study progressed.

September 2000

Prior to the beginning of the school year, all four levels of approval had been secured for the thesis study. The university Ethics Committee, the local board of education, the secondary school principal and the department head of mathematics all had endorsed in writing, and had shown interest in, the research. Letters of Informed Consent were distributed to the 52 students in the two Grade 9 Academic classes on September 14th.

As a result of an irate letter written by a concerned parent, the research was temporarily put on hold. In consultation with my advisor, the university chair of graduate studies, and the principal of the secondary school, a letter of response was drafted and returned to this individual, further explaining the research and its connection to the provincial curriculum. Although the parent decided not to grant permission for the participation of his child in the formal aspects of the research, no further correspondence regarding his various objections materialized.

These events had a notable, two-fold effect. First, after the shock of the original letter's contents had passed, I found myself deeply focused as I prepared the response letter defending my research in both its purpose and form. Second, the rough draft of the first assignment was meticulously reconstructed using the language, expectations, and divisions found within the provincial mathematics curriculum document (OMET, 1999). The unfortunate cause had produced most beneficial effects. Four other students did not return permission forms, making the total participants number 47 out of the 52 Grade 9 students in both classes.

As a prelude to the first assignment, the video entitled *Donald Duck in Mathemagic Land* (Disney, 1960) was shown to both classes, as were student examples from the Grade 11 pilot study. The central bulletin board featuring the title banner *Learning Between the Lines* (see Appendix O, CD-ROM) and other materials pertaining to the first assignment were pointed out to the students. The first assignment (see Appendix E) and corresponding background information regarding the golden section were handed out and presented to the classes on September 28th. The due date for the first assignment was set for October 30th, giving students just over a month to complete the integrated project in its entirety. The students seemed both interested and anxious regarding the work set before them. In the field notebook I make mention that I am relieved and excited to finally get the research underway.

October 2000

During the first week of October I taught a lesson regarding the methods, both algebraic and geometric, of constructing the golden section rectangle. As a result of a shortened daily schedule, and without any preconceived plan, I fortuitously asked one of the Grade 9 classes to work in small groups on their first assignment to fill in some time. These groups were not assigned, and students also had the choice of working independently. Most chose to work with a partner or with a small group of friends.

This serendipitous event inadvertently added another dimension to the integrated project. The co-operative group work session, which was repeated next day with the other class, and then routinely as part of the subsequent assignments, facilitated quality communication and investigation. Students discussed the definitions, thinking/inquiry problems, and ideas for their final SMARTWORKS. Students could also be seen working in pairs or small groups as they measured blackboards, desk tops, posters, windows, money, and even themselves (classical Greek aesthetics question) in determining dimension ratios and their proximity to the golden section ratio. Although I had done group work on several occasions with students in the past, I had simply not conceived this idea as part of the integrated research, and therefore was pleasantly surprised with the results.

Several students brought their SMARTWORKS in early to show me. Some had done Internet research and wanted to share their lists of related sites. Others came after school to discuss ideas that they had been working on alone, or with the help of a parent/guardian. All of these events were encouraging as I witnessed students becoming more engaged with the assignment as the weeks passed.

Parent's Night occurred on October 19th. As a mathematics teacher, and especially as a Grade 9 teacher, I can usually expect a high number of people to attend this type of evening. This night was no exception. I was curious to find out what parents/guardians would have to say about the assignment and the research in general. The comments were varied and sparse. Parents/guardians were more concerned, at this stage, with their child's ability in mathematics, as opposed to the nature of one particular assignment that had yet to be completed. Some did express a genuine interest in this unique approach to mathematics education. A few were skeptical and wanted to be reassured that the curriculum was being covered properly.

During the last few weeks of October, guided by the insights that I had gained from the first assignment's highlights and imperfections, I finished work on the second assignment. Unlike the first and third assignments in which the final projects (golden section rectangles and tessellations) were common features of other integrated curricula, the second assignment proposed something quite unique – the interplay of Cartesian Plane analytic geometry with linear perspective drawing. There was potential for interesting results, and I was curious.

The first round of presentations began on October 30th. After a brief class discussion regarding rhetorical skills, students proceeded one at a time to the front of the room with their SMARTWORKS, stood atop a small platform, and presented their projects to their classmates. Three different video cameras recorded these events, adding to the excitement and related trepidation. I mention in the field notes how impressed I was with the wide variety of creative interpretations (see Appendices O and P, CD-ROM and Video) – from intricate, two-dimensional rectangles to complex three-dimensional designs including golden boxes, a mobile, and a Rice Krispies tower with added golden spiral.

A mysterious visitor appeared in our mathematics classroom the following day (Halloween, October 31st). The fifth century B.C. patriarch of both mathematics and music, Pythagoras himself – complete with a hooded robe, sandals, tunic, and the Pythagorean Brotherhood's secret mathematical pentagram emblem stamped on his palm – brought the first assignment to a most duly Grecian conclusion.

November 2000

Upon completion of the presentations, three things were done to the SMARTWORKS for each assignment. Each piece was digitally photographed against a colour background, assessed according to the mark allocation in the assignment, and then mounted as part of a classroom exhibition. For the first assignment, a large banner with the words *Gallery Golden* was displayed

with the student work along the top of one wall. Documented in the field notes is the discovery that duct tape, although possessing many useful applications, is an ineffective means of adhering projects to a concrete surface. The majority of the works were in the process of falling down the morning after they had been mounted. A product called Fun Tack was found to be the only method of securing large, heavy posters for extended periods of time. I purchased and tacked veritable mounds of this material by the time the research was complete.

On November 6th students watched the *Masters of Illusion* (Harper, 1991) video and the second assignment was then distributed and discussed. Two comments are recorded from this day, both made by relatively weak mathematics students. One asked, “Do we get to do another SMARTWORK?” and the other stated, “I think this one’s going to be awesome!” These were encouraging words at the time, especially since the second assignment was more detailed and prescriptive, causing many students to feel anxious, according to the questionnaire.

The exhibition of student work became a vital piece in the integrated puzzle. An entry from the field notes reads, “The visual display brings a great sense of completion and closure to the process – we have come this far together, and here is the evidence.” Students not only liked seeing their works on display, but also enjoyed the mixture of student works from both classes in the makeshift gallery. The action of displaying all student work, as opposed to only the finest works, is an important and recurring theme in both the field notes and later in the interviews. One student described a related experience in the following way:

“In elementary school there was something called a Principal’s Pride Board, and the teacher would only choose – say you did a math test or a spelling test and you got perfect or one wrong – then that went up on the Pride Board. But then the other kids that got maybe three wrong didn’t get their work up on the board. Here, everyone gets it up there so everyone has an

opportunity to see their stuff on the wall. It's nice. . . . Some kids, when they're younger, they stop trying because they know that they're never going to get it up there."

Letters were sent to the department head, principal and board superintendent, updating these individuals on the progress of the research and inviting them to come and see the student work. Some were able to attend.

In mid-November I was able to attend the Arts Education 2000 conference in Toronto, which for the first time brought together all four of the major arts disciplines – drama, music, dance, and visual arts – in one setting. I participated in several workshops including one on tessellations and another on golden rectangle construction. Several excellent resources were also purchased or noted, adding to my steadily- expanding library of integration ideas. The conference subtitle, "A Renaissance for the Information Age," seemed remarkably congruous with the second assignment.

Throughout the week that followed, several interesting interactions with students took place. A repeating student with special needs handed in the first assignment two weeks late. This was quite unexpected and his final project, of which he was very proud, was intriguing to say the least. Based on a potato chip company's small promotional product, this student constructed a golden section rectangle that folded in such a way as to always return to the original configuration after several consecutive folds. He saw this as a direct and visual connection to the notion of infinity, which is paralleled in the repeating patterns found within the construct of the golden rectangle. What this student lacked in punctuality and thoroughness, he more than made up for in perspicacity.

Three other students approached me separately during this week with basically the same question, "When can we learn slope and equations, so that we can start the SMARTWORK?" Unlike the first and third assignments, the second created a need for learning that preceded the

teaching of the related curriculum expectations. This served to heighten the student anticipation, and relates to the Constructivist paradigm in education that will be discussed in the next chapter.

On the following weekend, I once again travelled out of town to participate in both the EQAO training for the provincial mathematics assessment administration, and in the *Exemplars* train-the-trainer sessions. Both of these experiences proved useful with regard to the research. The former provided me with a small window on provincial assessment, as far as how the provincial test would be structured and assessed. It also brought to my attention the growing interest in performance-based investigations that were about to be piloted provincially, and which shared components with the research assignments (e.g. group work, written communication, creative application). The most critical piece obtained from the *Exemplars* training was the instruction on creating meaningful rubrics that tied directly to the provincial curriculum. I would use this skill in the weeks to follow as I developed my final assessment rubric for the research (see Appendix H).

While away, I had purchased yet more resources. One was of particular interest to the students as I shared it with them on the Monday that I returned. Entitled *Leonardo da Vinci* (Corbis, 1998), the computer software CD-ROM featured multi-media, in-depth analysis of this artist/scientist, the Renaissance era, and the 32-page sketchbook – the Codex Leichester – that Bill Gates had purchased in 1994 for the formidable sum of $30 million (U.S.). I also showed the students two examples of backwards handwriting which had been given to me by two left-handed colleagues that were actually capable, like Leonardo in his sketchbooks, of performing this skill. In fact, it is proposed in the field notes that, alongside the singeing of paper edges, this mirror writing component may have been the most exciting feature of the second assignment for students.

Unlike the first assignment, before which I was able to show many student samples, the second assignment, due to its novelty, had no available examples of student SMARTWORKS. Therefore, I went about creating one myself. This experience proved to be very refreshing as the researcher was able to fully participate in the excitement of the creative application process. Although the example ended up being quite complex, I wanted to demonstrate the many different possibilities that students could explore in their project, while still achieving all of the required elements. The interviews later revealed that this teacher example was somewhat overwhelming and that many examples at different levels of complexity would have been much more helpful as they were in the first assignment. This of course would now be possible in future administrations of the assignments.

It should also be noted that new technologies such as TI-83+ graphing calculators, motion detectors, and *Geometer's Sketchpad* computer software were all introduced to my students, in conjunction with the regular textbook instruction, throughout this section of the course.

As November drew to a close, the third assignment was feverishly completed during late nights and weekends. Because certain necessary elements of the curriculum had not yet been taught (e.g. slope and equations), the original due date for the second assignment was postponed from December 4th to 18th, allowing students sufficient time to both absorb and apply the new learning.

December 2000

The month of December featured the following highlights: several new ideas, the second set of presentations, a fabulous pun, a revelatory mistake, changes to the instruments, and a sickening feeling likely akin to that of drowning.

New ideas were noted that both pertained directly to the research, and also were tangent to it. The notions of character studies, integrated curriculum writing guidelines, and a

parent/guardian Open House are proposed in the fieldnotes during this time. Likewise, the concepts of an ongoing "SMARTPAD" that would function as a Leonardo-type sketchpad and diary for students; and an "Art, Math, and Culture" field trip to Italy and Greece wherein students could experientially reinforce this rich web of integrated learning.

The presentations of the second assignment occurred on December 18th. Due to the prescriptive nature of this SMARTWORK, the final products were relatively similar in appearance and content, which led to somewhat repetitive presentations. Though perhaps less visually stimulating, I note that the second assignments smell wonderful (coffee and tea stains) and that I believe they likely reinforced the new learning better than the first. One student, in describing an unfortunate circumstance that had occurred while he was singeing his project, mentioned the words, accident and wax. In an attempt to rekindle the class spirit, I officially informed everyone that in educational circles this was formally known as a "waxident." Some laughed – most groaned.

The third and final assignment was handed out the following day. Some students were concerned with the limited amount of time left in the semester, the approaching provincial assessment, and the intricate nature of the tessellation project. Although I too shared these very sentiments, I informed them that including the break, there was still over a month left to complete the final assignment. I also added that even with six weeks allotted for completion, as was the case with the second assignment, some students still procrastinated until the last few days.

The revelatory mistake that was referred to above denotes the decision to let students take the second questionnaire home. Although this was intended to allow more time leading to more thorough responses, relatively few of these questionnaires were ever returned. Beyond this, I knew that the questionnaire itself was inadequate and that its very structure and content needed to be overhauled.

Both the students and I were growing tired and longed for the break. Earlier in the month, after a wakeful night of daughter noises, and following a less-than-productive group work session, I made the following comment in the field notes: “I feel just terrible and should go and eat lunch. On a day like today, you do sometimes wonder why you even bother trying something original and different.” Just before the break, signs of fatigue were again evident in the following entry, “Keep going Dan. If nothing else, you will have attempted something new.”

In consultation with my advisor and the Ethics Committee, several key changes and additions were made to the instruments. Along with corresponding rationales, these changes included: (i) a completed rubric for final assessment, (ii) a new quantitative final survey/questionnaire, (iii) the idea of an Open House with accompanying letter and response form, (iv) a rewording of interview questions, and (v) the addition of Grade 11 interviews.

January 2001

On January 9th the interviews began with all available Grade 11 students from the pilot study. As mentioned in previous chapters, the results from these interviews were overwhelmingly positive. By January 10th, the second assignments had been digitally documented, assessed and mounted in a second student exhibition, this time with a title banner reading *The Linear Louvre*. Empty wall space in Room 202 was now becoming hard to find.

Due to the size of the large sheets of grid paper on which the students had completed their second SMARTWORKS, I originally hung only the best dozen or so works. Immediately, however, I began to hear comments like, “I guess yours wasn’t good enough to make it up there next to mine.” and “Where are the rest of them?” Consequently, I once again visited the local office supplies depot and depleted their stock of Fun Tack. Nearly every available wall space, even *below* the bulletin boards, was now full. All 44 of the submitted final projects were on public display and students were content.

All except one, that is. This student had every right to be upset, and I could totally understand the frustration that prompted her to compose the anonymous note, with letters clipped from various magazines, demanding the soon return of her masterpiece. Since I was severely conserving Fun Tack by the time I had hung her creation, it consequently pulled loose from the wall, ended up on the floor, and due to its scrunched, dirty Renaissance appearance, was transported by a janitor to a nearby trash can. Fortunately, this student's work had been photographed, and so, when she was presented with a large colour copy of her misplaced project, her sense of humour prevailed and all was forgiven.

The provincial mathematics Grade 9 assessment took place in my classes from January 16^{th} to 18^{th}. Although this schedule essentially cut two weeks out of the regular teaching calendar (i.e., the week of, and the week following, the test – since it was a summative assessment), the accelerated delivery of the final pieces of the curriculum did procure for me those few key days during the last week of classes wherein I was able to finish the final presentations and administer the survey/questionnaire.

The final presentations transpired on January 22^{nd}. These colourful, transformational geometry creations were suspended from the ceiling using clothespins on two long pieces of rope suspended from the corners of the room and crossing in the center. The title banner hanging near the entrance read, *Escher Exhibition*. Once again, the student work was quite varied and included patterns made from squares, triangles, and hexagons, as well as a T-shirt design, a photograph puzzle, a butcher paper necktie, two tessellated homemade guitars, and a hanging collage. Segments from two newly-acquired Escher videos were shown after the presentations, and students later noted that they enjoyed seeing clips of Escher, Penrose, and Coxeter, even though they were shown later than usual, with regard to the introduction of this particular assignment.

As described in detail earlier in this chapter, the Open House on January 23rd went very well. On January 24th, the final survey/questionnaire was administered. During the previous week, a schedule had been distributed, and both individual and group interviews were already well underway after school, during lunch periods, and in the mornings. Invitations were extended to the superintendent, the principal, the department head, my advisor, and several other colleagues who I thought might be interested in witnessing this ordered chaos of student work before it was taken down. Some were able to attend and enjoyed the show.

The following quotation culled from the field notes aptly expresses the mixture of both my stress and my relief, as all of the above pieces systematically fell into place throughout the final month of January. “This semester has truly been five months of memorable education as a practitioner.”

In the weeks that followed, the SMARTWORK exhibitions were dismantled, the Fun Tack was reassembled into a giant mass, the assignments were sorted and assessed using the rubric, the interviews were transcribed, the survey results were entered into a spreadsheet for analysis, and the researcher began the daunting task that now lay ahead of him.

Chapter Five: Analysis of the Findings

Analysis of the Findings from the Instruments

Much has been written about the Constructivist paradigm in education, in which students construct their own meaning or learn through a variety of classroom strategies. Fogarty (1999) claimed that each of the following constructivist theorists of renown offered something unique to the implementation of a powerful and meaningful curriculum: Dewey, Piaget, Vygotsky, Feuerstein, Gardner, and Diamond. She summarized their various models and semantic contributions,

> Notice the elegant structures for intellectual challenge in problem-based learning models; exquisite moral dilemmas illuminated in the case study approach; splendid thematic units that thread concepts and skills across the disciplines; magnificent collaborative problem solving in robust project-based units of study; magical personal and interpersonal transformations through service learning curricula; and memorable moments of peak learning experiences through student-initiated and student-executed performances of the heart. (p. 14)

This research study, based on the integrated assignments, featured several of the above-mentioned activities. The Grade 9 students in my two mathematics classes experienced thematic units, collaborative problem solving, and peak learning experiences within the parameters of this secondary school case study. Both the qualitative research instruments, such as the questionnaires, interviews, Open House, and field notes, and the quantitative final survey revealed patterns regarding student perceptions of an integrated curriculum. These patterns, as documented in the previous chapter, supported the idea that meaningful educational experiences, perceived as reinforcing the new learning, were the direct result of these types of Constructivist teaching and learning strategies.

Analysis of the Findings Related to the Research Questions

First, it is interesting to note that the survey revealed that Grade 9 students tended to favour the more social activities such as cooperative learning, exhibition of student work, and the Open House evening, over the more individual activities of Internet/home research, *Geometer's Sketchpad* connections, and oral presentations. Second, the findings from the instruments, having been presented separately and in considerable detail in the previous chapter, will now be analyzed in terms of the five supplementary research questions.

The Communication of Mathematics

Having had the opportunity to mark both the integrated assignments and the provincial mathematics assessment written by my students in January (teachers were permitted to mark student tests prior to returning the packages to EQAO), it became quite clear to me that an overall weakness existed in my students' ability to communicate their mathematics understanding. Notwithstanding the capable work of some the stronger students, the general inability to translate mathematical algorithms and problem-solving strategies into coherent, full-sentence solutions was painfully evident in both the integrated assignments and in the various sections of the provincial test.

Therefore, although the integrated assignments certainly did not cure students of this inadequacy, and even though students did rank written communication last regarding their favourite activities, the survey did show that 68% of participants either agreed or strongly agreed that "the integrated assignments helped them to better communicate (written and verbally) the new math learning." Furthermore, the questionnaires and interviews with both the Grade 9 and Grade 11 students revealed that the majority of students found that the "communication (written and oral) of the projects helped them to better understand the various math concepts." The following are a few relevant quotations from the instruments regarding the perceived

improvement of mathematics communication skills and the related reinforcement of the new learning.

"Yes, it [mathematics communication] solidified it for us – that you had to make sense to yourself in writing and then make it so that everyone else could understand you, made it kind of easier to understand."

"It did, actually [reinforce new learning]. If you make something, that's one thing. But if you write about it and analyze it, you get a whole lot more out of it."

"Yes, it [communication] made you have to know what you were talking about."

"You explain it and then others explain it – so you take yours and theirs and put it together and understand it a lot more."

In answer to the original question, and in light of all the data gathered from the research instruments, it is my considered opinion that the segments of the integrated curriculum pertaining to the *Communication of Mathematics* were perceived by students as having somewhat reinforced the new mathematics learning.

The Creative Application of Mathematics

When asked on the survey if the "three integrated assignments helped them to creatively apply the new mathematics learning," 85% of participants responded as either agreeing or strongly agreeing with this statement. When further asked in the questionnaires and interviews if they felt that the "creative application of the new learning, as experienced in the assignments, helped them to better understand the new mathematics concepts," the responses from the majority of Grade 9 and Grade 11 students were affirmative.

The following quotations from the research instruments give examples of these types of student perceptions.

“These assignments helped me to creatively apply the new learning because I understand things better if I can do, see, and learn about the topic – then put it down on paper.”

“I think so, yes, because it showed you how to do it in different ways. The SMARTWORK – it showed you how to do it and what we were doing in class did too, so it gave you different aspects that you could put together.”

“Yes, when you do something creative and visual, you remember it more easily.”

In answer to the original question, and in light of all the data gathered from the research instruments, it is my considered opinion that the segments of the integrated curriculum pertaining to the *Creative Application of Mathematics* were perceived by students as having considerably reinforced the new mathematics learning.

The Effects on Student Motivation

Student perceptions varied greatly on this question. First I will share some comments made by students that demonstrate this variation, and then I will summarize from the teacher’s perspective.

“I think it motivated me a little bit more, but I really hate art. I’m not sure why – probably because I’m not very good at it. But it was a nice change from the normal textbook dry stuff.”

“Well, I don’t think it affected my interest in it [mathematics course] at all. I like art, I like math, that was a nice combination. I enjoyed it but it didn’t affect my motivation towards anything any different.”

“I think it motivated me a lot to try harder because if you put both the creative aspects and the actual theories together it makes more sense. You can actually use it and apply it to something.”

“They motivated me a lot because I really like doing stuff like that and I’m going to really miss it next year. They were fun.”

The survey revealed that when asked if the three integrated assignments increased their motivation for learning in the course, 48% of participants agreed with this statement and 16% noted that they strongly agreed. So although the opinions varied in the interviews and questionnaires, a combined 64% of Grade 9 students felt that the integrated assignments increased their motivation in the course. If this statistic was combined with the Grade 11 responses, from students who had experienced many secondary school courses and therefore had an increased ability to form meaningful comparisons with other courses regarding motivation, the resulting percentage would be much higher still.

I have now taught the Grade 9 academic mathematics course twice without the integrated assignments, and as many times with them. From the instructor's point of view, even when considering the exciting new technologies (e.g. graphing calculators, motion detectors, software, Internet) that have been used in all four courses, I would say that students were generally more motivated as a result of the integrated learning. There is no doubt that this was partly due to the new energies that I brought to these courses, but I believe it was also a result of the way in which the assignments engaged the learners at a personal and meaningful level.

In answer to the original question, and in light of all the data gathered from the research instruments, it is my considered opinion that the integrated assignments were perceived by students as having enhanced their motivation in the course.

The Qualities of the Integrated Learning

This research study also sought to define some of the qualities germane to the integrated learning process. I will briefly articulate three such qualities that have become apparent throughout the research process; complexity, playfulness, and universal appeal.

Complexity. Sylwester (1995) hypothesized that a brain-based curriculum of the future might "increase the importance of such subjects as the arts and humanities, which expand and

integrate complex environmental stimuli, and reduce the importance of basic skills and forms of evaluation that merely compress complexity" (p. 23). While the integrated assignments were designed to reinforce expectations from the curriculum, they did so in a way that required students to both process new information through a variety of senses, and then creatively apply the new learning through complex expressions of communicated understandings.

This non-traditional form of mathematics learning is not only unfamiliar to the students, but can also be terrifying for the educator. Wiggins and McTighe (1998) explained this phenomenon:

> As teachers, we are trained to believe our job is to remove doubts and explain things. Teaching is meant to take away the deadends, false starts, and surprises of inefficient inquiry. But teaching engagingly and effectively for understanding often requires that we persist in asking questions, delaying or avoiding giving answers, confronting students with problems, and putting mysteries and the need to rethink things constantly before them. (pp. 141-142)

When asked during interviews if a math/art course, based entirely on integrated units of study, would be a good idea, many students supported such a concept. With regard to interdisciplinary educational complexity, the following student verbalized a common attitude that surfaced several times and in various forms within the interviews and questionnaires:

"I think that it would probably enrich the learning as opposed to distracting from it, because school is combining everything you learn and applying it to the real world. You take things from one course and stuff from another course and hook them together."

Conspicuously evident within the fabric of the assignments themselves, and permeating the processes of comprehension, creation, and communication that each student underwent, complexity was recognized as one of the unique qualities germane to the integrated learning process.

Playfulness. Another quality that surfaced through daily observations and the research instruments was that of playfulness, or the sense of fun or festivity experienced by students within the integrated learning. Pedoe (1976) referred to this quality as what he called the pleasure principle, and encouraged practitioners to "consider this principle of fundamental importance in all education, and especially in mathematical education" (p. 13).

Davis and Rimm (1994) described the elements of humour and playfulness that figure into the character traits of the creatively gifted student:

> An especially frequent trait of creative students is a good sense of humor. Humor is first cousin to the ability to take a fresh, childlike, and playful approach to problems. Many discoveries, inventions, and artistic creations are the result of 'fooling around' with ideas and playing with possibilities, unconstrained by well-learned habits, traditions, and conformity pressures. Both Carl Rogers and Sigmund Freud agreed that regression to a more childlike, fanciful, playful state of mind is an important feature of creative thinkers. (p. 35)

Some of the more insignificant activities involved in the assignments such as the measuring of the proportions of student volunteers (i.e., full height vs. height to navel) in front of the class, the impromptu lesson on 'how *not* to singe one's Renaissance drawing,' or the hands-on template construction as an introduction to tessellations; these were, ironically, the types of moments that students indicated were perhaps most memorable. They certainly featured elements of playfulness, and were only possible within a learning environment where students felt safe enough to explore, laugh, and admit that learning can be fun. What follows are three student quotations along these lines:

"I realized that it could be exciting and fun to play with the concepts, and manipulate them however you saw fit. I enjoyed this project because it caught my interest, and if more people get excited about the project, as I did, it could be beneficial to do every year."

"It's interesting learning where math fits in to everyday life. It's also neat to see the other parts of math that you don't usually see in class."

"I liked how the SMARTWORKS brought two things together – it was fun and you were learning too."

The notion of playfulness ties directly to student motivation, which has already been discussed at length in this chapter. Gardner (2000) noted the connections between fun, pleasure, play and motivation in the following:

> Flying in the face of the behaviourists, who tied motivation directly to the receipt of tangible rewards, researchers now believe that learners are best served when their motivation is intrinsic: when they pursue learning because it is fun or rewarding in itself, rather than because someone has promised them some material benefit. (p. 76)

Therefore, playfulness, as encouraged through the assignments and as found to be a motivator for both student and teacher alike, forms a second quality germane to the integrated learning process.

Universal Appeal. A third ostensible quality of the integrated learning context was that of universal appeal. The survey revealed that when asked if this integrated approach to mathematics education, connecting it to visual arts, should be used regularly, 83% of respondents either agreed or strongly agreed. The questionnaires, interviews, and parent/guardian response forms all lead me to believe that the vast majority of students liked the assignments. As evidenced in the character studies earlier in this chapter, different students liked the integrated learning for many different reasons. For the gifted, or if not formally recognized as such, the brightest mathematics students, these assignments provided an ambiguous space in which they could explore and express themselves within flexible parameters. Davis and Rimm (1994) described the common learning styles of these types of students,

> Gifted students tend to be independent self-motivated learners more than teacher-motivated. They need and enjoy learning tasks that are unstructured and flexible, rather than the highly structured tasks needed by less able students. They prefer active participant approaches to learning, rather than spectator approaches; and they have 'well-integrated perceptual strengths,' meaning that they can learn through varied sensory channels, including auditory, visual, tactile, and kinesthetic. (p. 32)

For the average mathematics student, the assignments offered alternative means for demonstrating their understanding through forms of communication and creative application that may not have been otherwise available through more traditional assessment methods. Although the act of infusing visual art into the mathematics curriculum was not envisioned as a means of making the achievement of higher marks easier, both weaker and average students did express the opinion that they felt that their achievement in the course had improved as a result of the variety of assessment strategies.

The scope of the assignments was broad enough and the time frame long enough that students of all levels of ability could often capitalize on their personal assets through their selection of SMARTWORK ideas, and minimize their liabilities on other sections via cooperative group work, parental and teacher support, and personal research. In a related vein, Gardner (2000) noted that his Multiple Intelligences Theory tied motivation to existing talent.

> The theory of multiple intelligences suggests another factor: people may be most motivated to learn when they undertake activities for which they have some talent. In pursuing such activities they are likely to make progress and avoid undue frustration. (pp. 76-77)

I believe that the quintessential secret of the integrated learning process lies in its ability to strike a successful balance between the reinforcement of curricular expectations and the release of the individual imagination. This balance may very well account for the quality of universal appeal that integrated learning apparently holds for students of different gender, ages, and mathematical ability. In answer to the original question, and as explained in the above passages, the following unique qualities were therefore found to be germane to the integrated learning process: complexity, playfulness, and universal appeal.

The Patterns Arising from Assessment

The fifth and final supplementary question from the research study addressed the recognition of patterns from within the assessment. In light of the assessment of student assignments by both rubric criteria and by Achievement Chart categories, the following patterns emerged: (i) overall weaknesses in the categories of *Mathematical Communication* and *Thinking/Inquiry/Problem Solving* (lowest achievement), and (ii) overall strengths in the categories of *Knowledge/Understanding* and *Creative Application of Concepts* (highest achievement).

The weaknesses in communication and problem solving, and possible reasons for their existence, have already been thoroughly discussed in this paper. The fact that the assignments only featured simple definitions, and no mathematics skill questions, in the category of *Knowledge/Understanding*, may partially account for the relatively high achievement in this category. When the first assignment was reworked for the Applied group in the semester to follow, questions regarding basic understanding of concepts were therefore added to this page, with the deletion of some of the pre-existing definitions.

One possible reason why students achieved highest in the *Creative Application* category, is that since the SMARTWORKS were voted as the most favourite criterion, students may have invested the most amount of time and energy in completing this aspect of the integrated assignments. Comments from the questionnaires and interviews, as well as my own interpretation, having marked all the various sections of the assignments, would certainly support such an hypothesis.

Analysis of the Findings Related to the Role of the Participant-Observer

Two areas of focus shall be dealt with in this section. First, I will attempt to briefly summarize the salient features of my "baptism-by-fire" experience with integrated curriculum

development. Second, I will share a few highlights regarding personal professional development that were made possible through this ongoing Action Research process.

On Integrated Curriculum Writing

Newton, in his imaginative explosion of ideas regarding the physical universe, was intimately familiar with the four modern elements of space, time, matter, and energy. Nearly three centuries later, Einstein paired this fundamental quartet with his theories of Space-Time Continuum and Energy-Mass Equivalency. As a tribute to both of these inspirational life-long learners, and begging the indulgence of the reader, I would like to hang the four *sine qua non* elements of integrated curriculum writing upon these historical posts.

Creative Space. The integrated assignment must be designed in such a way as to provide the student with sufficient space to creatively personalize the projects involved, according to the limits of the individual's imagination. Students in this research study found that whereas the first and third assignments allowed for this type of application in the corresponding SMARTWORKS, the second assignment fell short in this area, being too prescriptive and narrow.

Adequate Time. The integrated assignment must allow for an adequate amount of time in which students may consciously and subconsciously develop and wrestle with ideas. This struggle must transpire over a period of sufficient time in order for the work of students to fully emerge into its mature form.

By setting tentative due dates, allowing approximately one month per assignment, this invaluable time was procured in the research. Students indicated that, notwithstanding their tendency to procrastinate regardless of time allocations, they would not have wanted less than three weeks to complete each assignment. From the teacher's standpoint, this month-long time period, with carefully planned and placed group work sessions, computer lab investigations, art lessons, and teacher/student conferences, was also deemed appropriate.

Legitimate Matter. Above all else, the integrated assignment must feature meaningful questions or projects that tie directly to, and serve to reinforce, the related curriculum from both or all disciplines involved.

In Ontario, at the time of this publication, this referred to Overall and Specific expectations as detailed in the provincial curricular documents (OMET, 1999). The knee-jerk reaction to most integrated educational endeavors involving art is to suspect a lack of legitimate matter and to relegate the learning experience to a meaningless charade designed to increase motivation, fill time, or to merely entertain students. Accusations of this sort are simply not possible to substantiate if the integrated learning is conscientiously tied to the curriculum.

Activities that are perceived by students as "fun" are not always legitimate or meaningful (Franks, 1995). In fact, one of the greatest challenges faced by teachers is preparing activities that are at once highly motivational and also academically sound. From a mathematical and Constructivist perspective, Franks noted his related concern,

> If "fun" is a code word for finding mathematical experiences interesting and challenging, if it defines mathematical situations that induce conflict that students <u>want</u> to resolve through more investigation, than I am very supportive. My sense is, however, that claims of doing "fun" mathematics often imply that the mathematics is not challenging. (p. 83)

Regardless of which disciplines an integrated curriculum seeks to connect, it must strive to do so in such a way that the related curricular expectations are clear and justifiable. Only in this manner can increased student motivation be genuinely construed as legitimate.

Infectious Energy. The integrated assignment is most successful when powered by three sources of infectious energy: that of the preceding students, that of the practitioner, and that of the student participants.

As any art instructor who has kept, over the years, an ever-expanding portfolio of student work will know, the learner likes to see examples. A variety of examples, to be precise, that

demonstrate a wide range of ideas, ability and success. The questionnaires and interviews in the research study revealed that students harbour extremely different views on the exact number of examples that should be shown at the onset of a new assignment. Some argued that as many as possible would be appropriate to stimulate ideas. Others claimed that no, or relatively few, examples should be shown, as they tend to hamper creativity and actually retard progress. Having shown many examples for the first assignment, one example for the second, and no examples for the third, I'm convinced that the ideal strategy lies somewhere in between the two polar student preferences.

The second form of infectious energy comes from the instructor. The assignments must be presented in such a way as to fascinate and excite the student, using the greatest variety of methods and media available. For example, the integrated assignments in this research were introduced with videos, books, computer software, posters, student work samples, role play, and a violin. Ongoing enthusiasm regarding the new learning and integrated projects must be maintained, even during those times when it feels less than natural. This actually proved to be less difficult than it appeared, since the teacher was simultaneously rejuvenated by the third form of infectious energy, that of the students.

The first set of presentations was a mysterious, great unknown for both myself and the students. However, once they had experienced the somewhat magical atmosphere of an art critique environment; once they had shared in the excitement of not only explaining their ideas to their peers but also of witnessing a myriad of approaches to the same learning task, the students began to show signs of academic empowerment.

Objects in motion tend to stay in motion. Newton's first law, as applied to the above-mentioned energies that propelled the integrated system, literally came to my rescue at the end of the semester, as time ticked away and due dates drew nigh for both myself and the students.

On Personal Professional Development

Rubric Construction. The construction of the assessment rubric, patterned after those presented in the newly-released Exemplars document (OMET, 2000), involved a serendipitous harmony of non-related events. As mentioned in chapter 4, the Exemplars training that I received in November 2000, occurred at precisely the same time that I was attempting to rework the assessment of the integrated assignments. With the technique still fresh in my mind, and using both the selected expectations and the Achievement Chart categories, I began to construct the assessment rubric (see Appendix H) that would be used in the final, holistic assessment of the assignments.

Holistic Assessment. Even though I had taken part in the marking of the Grade 3 provincial mathematics assessment in 1997, I still had difficulty preparing and motivating myself to complete a holistic assessment of the integrated assignments. Considering the fact that I had already marked all 144 assignments by Achievement Chart category, and taking into account the accumulated stress and fatigue that accompanied the last few weeks of the semester, it is perhaps understandable why I almost did not bother.

However, because a more thorough assessment would potentially help both the students and the quality of the research reporting, I forced myself through the exercise. In retrospect, I'm very glad that this decision was made. Along with their regular report card, students from my two Grade 9 academic mathematics classes also received a rubric that highlighted their Level achievement on each of the seven criteria.

Digital Competency. The research study also allowed me, via necessity, to become more familiar with the following technologies: digital cameras, scanners, WinZip drives, Laptops, data projectors, and computer software such as Adobe Photoshop, Microsoft PowerPoint, Geometer's Sketchpad, Corel QuattroPro, and the finer intricacies of Microsoft Word. I have experienced and

enjoyed a major learning curve in these areas of professional development. This has increased my confidence and digital competence in the ever-changing information age and has hopefully, in a general sense, improved my teaching.

Chapter Six: Implications for Practice and Research

Conclusions

Posted within mathematics classroom 202 is my favourite educational quotation that has become a sort of personal watchword with regard to the integrated research and my own pedagogy. Einstein defined the practitioner's role in the following way, "It is the supreme art of the teacher to awaken joy in creative expression and knowledge." Herein, he not only encapsulated the logical and aesthetic duality of our calling – to teach basic skills and to spark creativity – but he reminds us of the true touchstone of all education – the joy of learning.

In drawing conclusions from the study, it would be prudent to return to the original research problem and questions. The former stressed the difficulty that students have in communicating and creatively applying their mathematics learning, and suggested that an integrated approach, borrowing concepts and practices from the visual arts, might potentially help to overcome these inadequacies. The research questions further elaborated on this hypothesis, by highlighting the investigation of student perceptions, qualities germane to the integrated learning process, and patterns arising from the assessment of student work.

This study has shown that the integration of the two disciplines, in the form of the three assignments, was perceived by students as having reinforced the new learning through the communication and creative application of mathematics, and as having enhanced their motivation.

The communication of mathematical understanding is becoming a vital focal point both provincially and internationally for mathematics educators. Craven (2000) summarized the conditions necessary for the nurturing of rich mathematical communication:

> Children must feel free to explore, talk, create and write about mathematics in a classroom environment that honours the beauty and importance of the subject. Children

> must be encouraged to explain their thinking. Teachers must construct tasks that will generate discussion and provide an opportunity for students to explain their understanding of mathematical concepts through pictures, words, and numbers. (p. 27)

Naisbitt and Aburdene (1990) predicted in *Megatrends 2000* that the 20th century would become known in retrospect as the Dark Ages, and that future generations would experience a modern Renaissance for the arts, in which the emphasis would switch from sports to the arts, and wherein the definition of basic education would be expanded (1995, p. 6-9). I sincerely hope that they are found to be prophets of accuracy. I also hope that other disciplines, such as mathematics, will be able to capitalize on this potential modern Renaissance by continually appropriating ideas and practices from the visual arts classroom, as was successfully attempted in the creative application of mathematics concepts within the integrated assignments.

The *Microsoft PowerPoint* slideshow presentation (Appendix O, CD-ROM) that accompanies this study presents a variety of student work from the assignments, juxtaposing visual, colour images of the SMARTWORKS with the corresponding written communication. The video documentary (Appendix P) highlights significant moments from the verbal communication of the above projects during the student presentations. I believe that both of these additions serve to support my conclusions, and I hope that they shall provide readers with more comprehensive insight into the nature and results of the research study.

The *Learning Between the Lines* research study does not showcase an educational panacea. As has been shown throughout the various chapters, the two disciplines of mathematics and visual arts share a long and rich history of philosophical, practical, and educational connections. My simple objectives have been to remind, to experiment, and to recommend.

Relevance to Educational Practice

Change can be an exciting, painful and lonely experience. I am reminded of Abbott's *Flatland* (1884) novel, in which Mr. A. Square hopelessly attempts to convince his family and society of the existence of a higher dimension, from which the visiting Sphere character has emanated.

The secondary school is very much entrenched in the disciplines. As Eisner noted nearly thirty years ago, "They say it takes a generation for an innovation in education to find its way into the average classroom" (1972, p. 48). At the secondary panel, the full-scale integration of visual arts and mathematics curriculum would require dedicated team planning, teaching, and assessment as well as a supportive administration and local board. The arts-infusion approach that was by necessity used in this study, could be implemented by any mathematics educator. This journey however, demands both dedication and a risk-taking attitude. As Wiggins and McTighe (1998) explained,

> Everything we have said about habits of mind, resistance, and courage applies not only to students but also to teachers. We resist change in our teaching style, but we must model overcoming resistance for our students. We use our preferred facets of understanding, but we must work on all of them for the benefit of our subject and our students. (p. 175)

In the elementary panel, arts-infusion approaches to curricular integration have already been happening in many classrooms for years. In order to facilitate more widespread implementation, professional development, such as the aforementioned *Beyond Stickmen* seminars or integration workshops, would be a necessity. In districts where art specialists exist in the elementary school, once again, cooperative teacher planning and assessment would be required.

Sylwester (1995) noted that, "It's interesting to muse on such widely acclaimed developments as thematic curricula, cooperative learning, and portfolio assessment. All require

more effort from teachers than do traditional forms of curriculum, instruction, and evaluation" (p. 24). This increased effort, required to cope with the discomfort of change and to venture beyond the boundaries of existing practice, is possible and profitable for the daring educator. In the words of Hargreaves and Fullan, "It is a passionate vocation. Good teachers are not just well-oiled machines. Computers can never replace them. They are emotional, passionate beings who fill their work and their classes with pleasure, creativity, challenge and joy" (1998, p. 53).

Limitations of the Study

There were two significant limitations of this study: the potential researcher bias and the logistical constraints on the original curricular vision.

As participant-observer, I did enjoy a unique and exciting vantage point. However, I was also forced to walk a metaphorical tightrope suspended between the realities of making the research happen and watching the research transpire. In this sense, much of the optimism that surrounded the integrated assignments and that is likely evident in their documentation may be accrued to my total absorption in the research instruments and their development. This is not to say that the enthusiasm was in any way counterfeit, or that it negatively affected the research. Notwithstanding the subjectivity inherently present in my personal decisions regarding the shaping, implementation, and assessment of the curriculum, by restricting the bulk of the conclusions to the statistical and recorded perceptions of students, and to the noted qualities of the learning, I am hopeful that much of the potential argument regarding researcher bias has been avoided.

The second limitation, as discussed in the research design section, was that a fully integrated mathematics and visual arts course was simply not possible. Such a course would have better facilitated the treatment of both disciplines as equal partners, and would have allowed the researcher to fully integrate the expectations from both Grade 9 courses of study. However, the

timetable, teacher course allocations, and the early preparation (e.g. Grade 8 sign-up in the spring of the previous year) required for such an idea, made this an impossible undertaking at this time. Therefore, arts-infusion was explored, rather than the preferred approach of a visual arts and mathematics fusion format.

Future Research Possibilities

In keeping with the spirit of Action Research philosophy, the "next step" in the research program was to plan for future action, based on the successes and failures of past experience. In the semester that followed the research study, the first assignment regarding the golden section was reworked, and redistributed to a Grade 9 Applied mathematics class. To complete the secondary panel investigation, another assignment was designed for my Calculus students entitled *Fluxions & Deductions: An Integrated History of the Calculus Assignment* (see Appendix N), which followed the same template pattern as those in the research study (i.e., organized by Achievement Chart categories and including a SMARTWORK project).

The first assignment that I would add to the Grade 9 triad would be one that grows out of the remaining curricular strand of *Relationships*, and would likely focus on statistical analysis with a potential tie to the medieval era and William the Conqueror and/or Hildegard von Bingen. Beyond this, I would someday like the opportunity to construct, orchestrate, and assess a fully integrated mathematics and visual arts course. Such a course, worth two credits in the secondary system, could take several forms.

It might echo the research found herein, as a series of 8-12 thematic assignments or units of study, each tied to an historical period, a particular character(s), and a set of expectations from both curricular documents. Or it might be more linear in nature, wherein the teacher would proceed through the traditional ten chapters of the mathematics text, and through the regular 8-10

visual art projects, and simply highlight ongoing connections between the disciplines throughout the school year.

Perhaps most exciting of all would be a format in which the curricular structure would cluster around regular field trips to art and science museums; explorations of local art, architecture, and mathematical phenomena; and a climactic class trip to Italy and Greece, the wellsprings of the disciplines of visual arts and mathematics respectively.

Aftermath and the Practitioner

The results of this Action Research study were presented at three distinct levels, throughout the remainder of 2001. Locally, I was asked to share the investigation with the Intermediate mathematics class of my advisor, in the teacher education program at Nipissing University. The *Learning Between the Lines* research study was also presented in May 2001 at the Ontario Association of Mathematics Education [OAME] conference held in Scarborough, Ontario. Following this, a co-authored paper, focusing on mathematics communication and featuring highlights from this study, will be presented in August 2001 (i.e., still future tense at the time of this publication) at an international mathematics educators symposium in Queensland, Australia entitled, *The Mathematics Education Into the 21st Century Project: New Ideas in Mathematics Education*. And finally, in conclusion, I am intrigued with the possibility of continuing the development of these ideas and this type of research at the post-graduate level.

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APPENDICES

Appendix A: Letter to Board of Education

Appendix A

Daniel H. Jarvis

Mr./Ms. _______________, Superintendent
School Board

Re: Thesis Research Proposal
August 2000

Dear Mr./Ms. _________________,

I am a teacher of secondary mathematics and visual arts at _________ . I am also a graduate student, currently enrolled in the part-time Master of Education program through Nipissing University, North Bay, Ontario. Completing my thesis is the final requirement for this program. I am requesting your assistance in helping me to finalize this thesis. The purpose of the study is to ascertain the effects of an integrated mathematics and visual arts curriculum on student achievement and attitudes at the secondary level. More particularly, I seek to understand if such an integrated approach to math learning would help or hinder students in attaining greater success in the *creative application* and *communication* of mathematics (Achievement Chart categories 3 and 4).

I have proposed three minor integrated assignments (*Greek Golden Section, Renaissance Perspective Drawings, and Escher Tesselations)* to be conducted during the regular program of the two Grade 9 Academic classes that I will be teaching in September 2000. These three assignments, along with an open-ended student questionnaire, random semi-structured interviews, and video footage of class presentations would be analyzed according to Glaser and Strauss' *Grounded Theory of Qualitative Research.*

I am asking for your consent to allow me to conduct this research, which has been approved by the Ethics Committee of Nipissing University, and has been discussed with the Principal and Math Department Head of _________. Informed Consent letters would be sent home

with students detailing the nature of, and confidentiality issues involved in, the research. These letters would also ask for parental/guardian permission to potentially use the anonymous results of the assignments, questionnaires, interviews, and video footage in the thesis report.

I have attached a copy of Nipissing University's policy on research ethics, as well as a copy of a sample questionnaire and interview format. Once again, I thank you for your assistance in this matter. Should you wish to discuss this study further, please do not hesitate to contact me at the above address.

I look forward to your reply.

Yours in Education,

Daniel H. Jarvis

Teacher and Student Researcher

Appendix B: Letter to Secondary School Principal

Appendix B

Daniel H. Jarvis

Mr./Ms. ___________________, Principal
Secondary School

Re: Thesis Research Proposal
August 2000

Dear Mr./Ms. ________________,

As a graduate student of the part-time Master of Education program through Nipissing University, North Bay, I am completing my thesis as the final requirement. As per our earlier discussions, I am requesting your assistance in helping me to finalize this thesis. The purpose of the proposed study is to ascertain the effects of an integrated mathematics and visual arts curriculum on student achievement and attitudes at the secondary level. More particularly, I seek to understand if such an integrated approach to math learning would help or hinder students in attaining greater success in the *creative application* and *communication* of mathematics (Achievement Chart categories 3 and 4).

After further preparations, and having conferred with my Thesis Supervisor, Dr. Douglas Franks, the final form of the thesis will be a qualitative case study analysis. I have proposed three integrated assignments (*Greek Golden Section, Renaissance Perspective Drawings,* and *Escher Tesselations)* to be conducted during the regular program of the two Grade 9 Academic classes that I will be teaching in September 2000. These three assignments, along with an open-ended student questionnaire, random semi-structured interviews, and video footage of presentations would be analyzed according to Glaser and Strauss' *Grounded Theory of Qualitative Research.*

I am asking for your consent to allow me to conduct this research, which has been approved by the Ethics Committee of Nipissing University and by the Superintendent of the Southern Region of ___________. Informed Consent letters would be sent home with students

detailing the nature of, and confidentiality issues involved in, the research. These letters would also ask for parental/guardian permission to potentially use the anonymous results of the assignments, questionnaires, interviews, and video footage in the thesis reporting. Final results of the study could be shared with staff in some format at your discretion, or as interest warrants.

I have attached copies of the following: (i) Nipissing University's policy on research ethics, (ii) Letter of Informed Consent to parent/guardian, (iii) sample questionnaire and interview format, and (iv) draft samples of the three assignments. Once again, I thank you for your assistance in this matter. Should you wish to discuss this study further, please do not hesitate to contact me at the above address.

I look forward to your reply.

Yours in Education,

Daniel H. Jarvis

Teacher and Student Researcher

Appendix C: Letter to Math Department Head

Appendix C

Daniel H. Jarvis

Mr./Ms. ___________________,
Department Head of Mathematics
Secondary School

Re: Thesis Research Proposal
August 2000

Dear Mr./Ms. ________________,

As a graduate student of the part-time Master of Education program through Nipissing University, North Bay, I am completing my thesis as the final requirement. As per our earlier discussions, I am requesting your assistance in helping me to finalize this thesis. The purpose of the proposed study is to ascertain the effects of an integrated mathematics and visual arts curriculum on student achievement and attitudes at the secondary level. More particularly, I seek to understand if such an integrated approach to math learning would help or hinder students in attaining greater success in the *creative application* and *communication* of mathematics (Achievement Chart categories 3 and 4).

After further preparations, and having conferred with my Thesis Supervisor, Dr. Douglas Franks, the final form of the thesis will be a qualitative case study analysis. I have proposed three minor integrated assignments (*Greek Golden Section, Renaissance Perspective Drawings,* and *Escher Tesselations)* to be conducted during the regular program of the two Grade 9 Academic classes that I will be teaching in September 2000. These three assignments, along with an open-ended student questionnaire, random semi-structured interviews, and video footage of class presentations would be analyzed according to Glaser and Strauss' *Grounded Theory of Qualitative Research.*

Working on the assumption that the three assignments will not detract from the rigorous provincial curriculum, but serve only to complement and hopefully reinforce it (proportion,

nature of the line, geometric patterning), I am asking for your consent to allow me to conduct this research within your department. The proposal has been approved by the Ethics Committee of Nipissing University, by the local superintendent of the Board, and by the principal of ________. Informed Consent letters would be sent home with students detailing the nature of, and confidentiality issues involved in, the research. These letters would also ask for parental/guardian permission to potentially use the anonymous results of the assignments, questionnaires, interviews, and video footage in the thesis reporting.

I have attached copies of the following: (i) Nipissing University's policy on research ethics, (ii) Letter of Informed Consent to parent/guardian, and (iii) sample questionnaire and interview format, and (iv) draft samples of the three assignments.

Once again, I thank you for your assistance in this matter. Should you wish to discuss this study further, please do not hesitate to contact me at the above address.

I look forward to your reply.

Yours in Education,

Daniel H. Jarvis

Teacher and Student Researcher

Appendix D: Letter of Consent to Parent/Guardian and Student

Appendix D

Daniel H. Jarvis

Re: Permission form for participation in
Learning Between the Lines: A Graduate Research Study

September 2000

Dear Parent or Guardian,

As part of my graduate studies at Nipissing University, I have prepared a research study in which an integrated approach, combining the disciplines of mathematics and visual arts, will be tested in two Grade 9 Academic mathematics courses. In particular, I seek to understand if such an integrated approach to math learning would help students in attaining even greater success in the *creative application* and *communication* of mathematics (Achievement Chart categories 3 and 4 of the *Ontario Curriculum* (1999). Your child's class has been selected to participate in this study.

Brain research and recent trends in provincial, national, and international assessment indicate that an integrated curriculum may potentially provide children with a more meaningful and powerful educational experience.

As part of the regular course of study your child will be completing three integrated assignments, each of which will focus on a specific topic from *The Ontario Curriculum, 1999* (proportion, the nature of the line, and geometric patterning).

Based on these assignments, I will be asking students to share their thoughts on a questionnaire, and will also ask several randomly-selected students to participate in short interviews. Since the current provincial math standards focus more heavily on *math communication*, I would also like to videotape the class presentations of the assignments.

Although the assignments will be mandatory and graded as part of the regular course of study, participation in the study (i.e., the questionnaire, interviews, and videotaping) is optional. If you and your child choose not to participate in the study, this will in no way affect the student's grade. All data that is collected will be coded as to provide for complete confidentiality being maintained with regard to the names of participants.

This study has been approved by the Ethics Committee of Nipissing University, the local Board of Education, and by the administration and math department head at _______. By signing this form you are permitting your child to participate in the study and allowing the researcher to use the assignments, questionnaire responses, interview comments, and video footage in the formal compilation and analysis of the research results.

I look forward with excitement to your child's continuing journey in mathematics education, and I anticipate a wonderful semester of rich learning experiences together. The results of this published thesis will be available for your reading in the near future. Please feel free to contact me regarding any questions that you as a parent/guardian may have at this time.

Yours in Education,

Mr. Daniel H. Jarvis

RESEARCH STUDY INFORMED CONSENT PERMISSION FORM

I hereby grant permission for the potential and anonymous use of my child's assignments and comments (written, audio, and video), in the reporting of this integrated research study. I have read the above letter and think that this study has been carefully prepared and approved, and therefore trust Mr. Jarvis to conduct his research in a professional manner in keeping with the conditions of the Ethics Committee of Nipissing University and the curricular standards of the Ontario Ministry of Education and Training.

[I] The Use of Comments and Class Assignments

Signature of Student: ___________________________________

Signature of Parent/Guardian: __________________________________

[II]The Use of Video Footage of Class Presentations

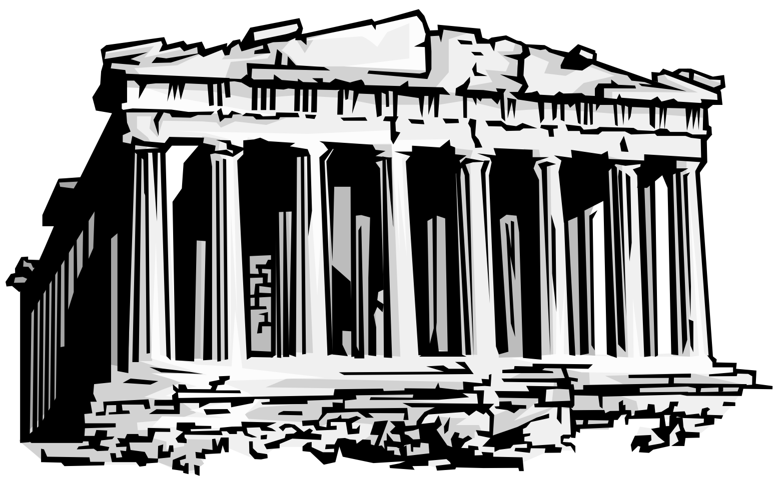
Signature of Student: ___________________________________

Signature of Parent/Guardian: __________________________________

Please note: The videotaping has been treated separately from the permission to use the other forms of data. Please sign both sections if you agree to have your child fully participate in the study, or only the first section if you do not want your child videotaped. Thank you.

Appendix E: Assignment 1—Ratio, Proportion, and the Golden Section

RATIO
&
PROPORTION



THE GOLDEN SECTION
ASSIGNMENT # 1

COURSE: MPM 1DA 01/02

DATE: OCTOBER 2000

STUDENT: ______________________________

RATIONALE AND RESOURCES FOR ASSIGNMENT # 1: THE GOLDEN SECTION

RATIONALE

Mathematical knowledge becomes meaningful and powerful in application. This curriculum embeds the learning of mathematics in the solving of problems based on real-life situations. Other disciplines are a ready source of effective contexts for the study of mathematics. Rich problem-solving situations can be drawn from closely related disciplines, such as computer science, physics, or technology, as well as from subjects historically thought of as distant from mathematics, such as geography or art. It is important that these links between disciplines be carefully explored, analysed, and discussed to emphasize for students the pervasiveness of mathematical knowledge and mathematical thinking in all subject areas.

The importance of communication in mathematics is a highlight of the elementary school curriculum and continues to be a highlight in secondary school. In all strands and all courses, expectations are included that require of students the clear and concise communication of reasoning or of findings. It is the expectation that students, over the course of their high school experience, will learn to write about their use of mathematics, effectively incorporating mathematical forms such as calculations, equations, graphs, or tables. This curriculum assumes a classroom environment in which students are called upon to explain their reasoning in writing, or orally to the teacher, to the class, or to other students in a group.

Excerpts from, *The Ontario Curriculum: Grades 9 and 10 Mathematics,* 1999, p. 4

NUMBER SENSE & ALGEBRA (p. 10)

OVERALL EXPECTATION

- By the end of this course, students will solve multi-step problems requiring numerical answers, using a variety of strategies and tools.

SPECIFIC EXPECTATION

- By the end of this course, students will solve multi-step problems involving applications of percent, ratio, and rate as they arise throughout the course.

SELECTED RESOURCES

Newman, R. & Boles, M. (1992). *Universal patterns: The golden relationship of art, math, & nature*. Bradford, MA: Pythagorean Press.

Ontario Ministry of Education and Training. (1999). *The Ontario Curriculum Grades 9 and 10: Mathematics.* Toronto, ON: Queen’s Printer for Ontario.

Runion, G. E. (1990). *The Golden Section*. Palo Alto, CA: Dale Seymour Publications.

MATHEMATICS KNOWLEDGE & UNDERSTANDING

Define the following terms: (1 MARK EACH)

$\overline{}$
10

RATIO:

PROPORTION:

THE GOLDEN RATIO (A. K. A. THE GOLDEN SECTION, GOLDEN MEAN, GOLDEN CUT, DIVINE PROPORTION, OR GOLDEN PROPORTION):

ϕ (GREEK LETTER & NUMERICAL APPROXIMATION):

PHIDIAS:

THE GOLDEN RECTANGLE:

THE GOLDEN TRIANGLE:

THE FIBONACCI SEQUENCE:

THE PARTHENON:

PENTAGRAM (A. K. A. THE PENTALPHA, PENTACLE, & PENTANGLE):

MATHEMATICS THINKING, INQUIRY, & PROBLEM SOLVING

Solve or complete the following questions:

___ / 10

CLASSICAL AESTHETICS (2 MARKS)

In ancient Greece, the classical sculptors frequently used the navel (belly button) as the Golden Cut of the human body. This was evident in their sculptures of the human figure (Kouros). How do you compare to the classical standards of beauty? In other words, is your navel the Golden Cut of your body?

A. Measure floor to navel = ___________ $\frac{B}{A} =$ _____________

B. Measure your overall height = ___________ My conclusion:

GOLDEN PROPORTION (2 MARKS)

Given a rectangle with dimensions 150 cm x 80 cm, determine if it is a Golden Rectangle (Show calculations). If not, determine what width would make it one.

THE ARTIST'S STRETCHER (2 MARKS)

An artist wants to design a canvas that is in the shape of the Golden Section Rectangle for her next large oil painting. If the maximum length possible for the stretcher is 240 cm, find the corresponding width (to 2 decimals). What will be the total perimeter of the stretcher?

THE GOLDEN HUNT (4 MARKS)

Analyze seven common rectangular objects around the house to determine if they are *golden.* Measure and divide the dimensions as shown.

OBJECT	LENGTH (B)	WIDTH (A)	RATIO (B/A)	CLOSE TO ϕ ?
Can. $10 Bill				

<u>MATHEMATICS APPLICATION OF CONCEPTS</u>

Create the following items:

15

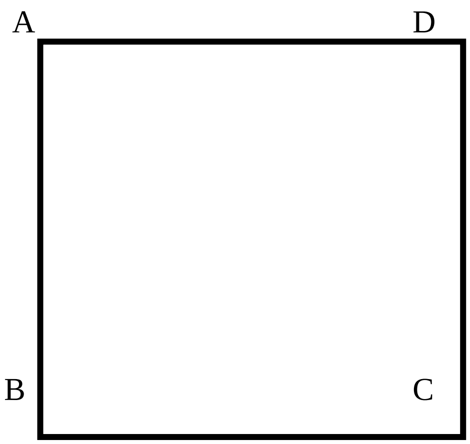
A GOLDEN SECTION (2 MARKS)

On the given line AB, label the Golden Section (GS) with a point.

A B

A GOLDEN RECTANGLE (3 MARKS)

From the given square ABCD, construct and label a Golden Section Rectangle.



A GOLDEN "SMARTWORK" (SECONDARY MATH AND ART WORK)(10 MARKS)

Based on the new learning from this assignment and using the handouts given, construct a SMARTWORK that features the Golden Ratio in some form. This project allows for great freedom of expression. Your piece may be 2- or 3- dimensional. It may be narrative (tells a story), literary (quotes or an illustrated poem), historical (perhaps a tie to math history?), abstract (no recognizable objects), a collage, a sculpture – your imagination is the limit!

You may choose to simply draw a Golden Rectangle, or you may choose to incorporate more complex ideas from any of the Golden Triangle, the Golden Spiral, the Pentagram, the Lute of Pythagoras, the Fibonacci Number Sequence, etc.

Once again, the final SMARTWORK <u>must</u> include the Golden Ratio somewhere in the composition and will also be assessed for creativity & complexity.

MATHEMATICS COMMUNICATION

Use mathematical language to communicate in the following forms:

15

WRITTEN DESCRIPTION OF YOUR "SMARTWORK" (10 MARKS)

Use the remainder of this page (add further sheets if necessary) to describe in detail your finished project. For example, you should explain exactly where the Golden Section(s) is located, the reason you chose this particular format, style, colour scheme, etc. Try to use as much mathematical language as possible (e.g. ratio, length, width, proportion, etc.).

VERBAL PRESENTATION (5 MARKS)

In a brief presentation to the class (3-5 minutes), simply display and describe your SMARTWORK. You may wish to discuss a few points from the planning stages (show preliminary sketches/ideas?) or from the process (changes made, new ideas, etc.). Relax. Enjoy the opportunity to share your creation with others.

<u>Resources for Assignment # 1: *Ratio, Proportion, and the Golden Section*</u>

Boles, M., & Newman, R. (1992). *The surface plane: The golden relationship in art, math and nature*. Bradford, MA: Pythagorean Press.

Devlin, K. (1998). *Life by the numbers.* New York, NY: John Wiley & Sons.

Disney. (1960). *Donald Duck in Mathmagic Land.* [Video-tape].

Frayling, C., Frayling, H., & Van Der Meer, R. (1992). *The art pack.* New York, NY: Alfred A. Knopf Publishing.

Ghyka, M. (1977). *The geometry of art and life.* New York, NY: Dover.

Huntley, H. E. (1970). *The divine proportion: A study in mathematical beauty*. New York, NY: Dover.

Ivins, W. M., Jr. (1946). *Art and geometry: A study in space intuitions.* Toronto, ON: General Publishing.

Lawlor, R. (1982). *Sacred geometry: Philosophy and practice.* New York, NY: Thames & Hudson.

Newman, R. & Boles, M. (1992). *Universal patterns: The golden relationship of art, math, & nature*. Bradford, MA: Pythagorean Press.

Pedoe, D. (1958). *The gentle art of mathematics.* New York, NY: Dover.

Pedoe, D. (1976). *Geometry and the liberal arts.* Markham, ON: Penguin Books.

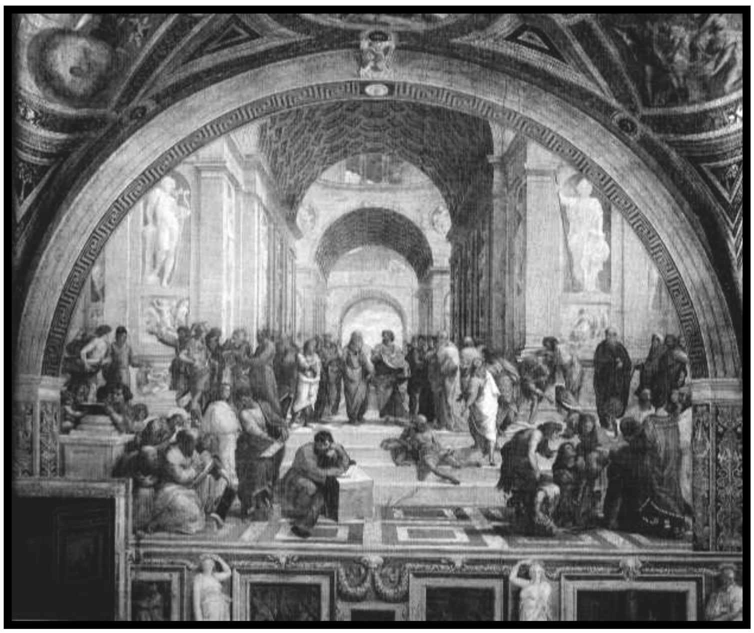
Runion, G. E. (1990). *The golden section*. Palo Alto, CA: Dale Seymour Publications.

Valens, E. G. (1964). *The number of things: Pythagoras, geometry and humming strings.* Toronto, ON: Clarke, Irwin & Co.

Wells, D. (1986). *The Penguin dictionary of curious and interesting numbers.* Markham, ON: Penguin Books.

Appendix F: Assignment 2—Linear Relations and Renaissance Perspective Drawing

LINEAR RELATIONS



RENAISSANCE LINEAR PERSPECTIVE ASSIGNMENT # 2

COURSE: MPM 1DA 01/02

DATE: NOVEMBER 2000

STUDENT: ___________________________

RATIONALE AND RESOURCES FOR ASSIGNMENT # 2: LINEAR RELATIONS

RATIONALE

Mathematical knowledge becomes meaningful and powerful in application. This curriculum embeds the learning of mathematics in the solving of problems based on real-life situations. Other disciplines are a ready source of effective contexts for the study of mathematics. Rich problem-solving situations can be drawn from closely related disciplines, such as computer science, physics, or technology, as well as from subjects historically thought of as distant from mathematics, such as geography or art. It is important that these links between disciplines be carefully explored, analysed, and discussed to emphasize for students the pervasiveness of mathematical knowledge and mathematical thinking in all subject areas.

The importance of communication in mathematics is a highlight of the elementary school curriculum and continues to be a highlight in secondary school. In all strands and all courses, expectations are included that require of students the clear and concise communication of reasoning or of findings. It is the expectation that students, over the course of their high school experience, will learn to write about their use of mathematics, effectively incorporating mathematical forms such as calculations, equations, graphs, or tables. This curriculum assumes a classroom environment in which students are called upon to explain their reasoning in writing, or orally to the teacher, to the class, or to other students in a group.

Excerpts from, *The Ontario Curriculum: Grades 9 and 10 Mathematics*, 1999, p. 4

ANALYTIC GEOMETRY (p. 14)

OVERALL EXPECTATIONS

- By the end of this course, students will determine, through investigation, the relationships between the form of an equation and the shape of its graph with respect to linearity and non-linearity.
- By the end of this course, students will determine, through investigation, the properties of the slope and y-intercept of a linear relation.

SPECIFIC EXPECTATIONS

- By the end of this course, students will determine the slope of a line segment, using various formulas.
- By the end of this course, students will plot points on the xy-plane and use the terminology and notation of the xy-plane correctly.
- By the end of this course, students will determine the equation of a line, given information about the line (e.g. the slope and y-intercept, the slope and a point, two points, a line parallel to a given line and having the same x-intercept as another given line).

SELECTED RESOURCES

IBM Canada Ltd. (1983). *I, Leonardo: A journey of the mind.* Educational packet. Markham, ON: Author.

Monk, C.H. (1975). *Leonardo da Vinci*. New York, NY: Hamlyn.

Ontario Ministry of Education and Training. (1999). *The Ontario Curriculum Grades 9 and 10: Mathematics.* Toronto, ON: Queen's Printer for Ontario.

Shlain, L. (1991). *Art & physics: Parallel visions in space, time and light*. New York, NY: Morrow.

MATHEMATICS KNOWLEDGE & UNDERSTANDING

Define the following terms: (1 MARK EACH)

/10

LINE:

LINEAR RELATION:

SLOPE:

THE RENAISSANCE:

LINEAR PERSPECTIVE:

VANISHING POINT(S):

HORIZON LINE:

LEONARDO DA VINCI:

CARTESIAN PLANE:

RENÉ DESCARTES:

MATHEMATICS THINKING, INQUIRY, & PROBLEM SOLVING

Solve or complete the following questions:

10

THE SCHOOL @ ATHENS (4 MARKS)

Research the famous painting found on the cover of this assignment and answer:

Artist: Find 2 Greek philosophers:
Date: Find 2 Greek mathematicians:
Medium: Find 3 Renaissance artists:

What is the juxtaposition of great philosophers & artists of the classical (Greek) & neo-classical (Renaissance) eras meant to symbolize?

On the cover, label the vanishing point (VP) and reconstruct at least 6 lines of linear one-point perspective.

SLOPE ON A ROPE (3 MARKS)

As the fortunate survivor of a tragic shipwreck, you find yourself alone on a deserted island with nothing but the clothes on your back and a long section of rope salvaged from the sunken ship. If the rope is conveniently knotted at every meter mark how could you calculate the slope of the leaning mast pole projecting from the beach? (Assume one can still climb it safely due to the depth of its sudden and forceful impact in the sand!)

SCOPING FOR SLOPE (3 MARKS)

Using a similar method (measuring tape), now calculate the slopes of three slanted objects from the real world around you. Record information in the following chart:

OBJECT	RISE (B)	RUN (A)	SLOPE (B/A)	SKETCH
Wheelchair Ramp	***0.7 m***	***2.1 m***	***1/3***	

MATHEMATICS APPLICATION OF CONCEPTS

INTRODUCTION: LEONARDO AND RENÉ

$\frac{\quad}{15}$

Suppose Italian Leonardo da Vinci (1452-1519) could somehow have crossed paths with French mathematician René Descartes (1596-1650). Suppose further that they had decided to collaborate on a series of special drawings that incorporated principles from both Renaissance linear perspective and 17th century analytic geometry. One wonders what kind of interesting results may have occurred & just how the two "sciences" would have related.

Create the following items using the sheets provided with the assignment:

A ONE-POINT LINEAR PERSPECTIVE DRAWING WITH SLOPES (2 MARKS)

- With the Origin (0,0) labeled as the Vanishing Point (VP), complete the construction lines necessary to visually render the illusion of depth for the 8 floating cubes. Remember to use vertical and horizontal lines to render all edges that are parallel to the edges of the frontal square. Estimate the proper depths to insure all sides appear equal.
- Calculate the slopes of the four construction lines indicated at the bottom of page.

A 2-POINT LINEAR PERSPECTIVE DRAWING WITH LINEAR EQUATIONS (3 MARKS)

- Using the x-axis as the horizon line and the two indicated coordinates as the two vanishing points, complete the construction lines for the floating cube and rectangular prism. Remember to use vertical lines to render all edges that are parallel to the frontal edge. Once again, estimate the proper depths of the objects.
- Calculate the slopes and linear equations for the four indicated construction lines.

A LINEAR "SMARTWORK" (SECONDARY MATH AND ART WORK) (10 MARKS)

- On the large sheet of graph paper provided, create a Renaissance linear SMARTWORK that encompasses all of the following items & conditions:
- Cartesian Plane (x and y axes) must be clearly labeled; not necessarily in center
- Rendered solids must include at least 1 cube and 1 rectangular prism
- Key coordinates (vanishing point(s) & vertices of solids) must be clearly labeled
- Construction lines (at least 4) must be labeled with corresponding equations
- Antiquated appearance: coffee (liquid) and dirt stains, crumple marks, singed edges
- Signature using Leonardo's backwards (mirror) writing style

The rendered scene may be interior, exterior, or fantasy. You may use either 1-, 2-, or even 3-Point Linear Perspective. It may be monochromatic (Burnt Sienna was quite popular in sketchbooks) or be rendered in colour. Further explorations might include adding an original invention sketch, experimenting with the shading of a sphere or human hand, personal notes or poetry, etc. The final composition will be assessed for the inclusion of the required components above, and also for creativity & complexity.

<u>MATHEMATICS COMMUNICATION</u>

Use mathematical language to communicate in the following forms:

___
15

WRITTEN DESCRIPTION OF YOUR "SMARTWORK" (10 MARKS)

Use the remainder of this page (add further sheets if necessary) to describe in detail your finished project. For example, you should discuss the slopes and linear equations of the relevant construction lines, the methods used to make the drawing look old, etc. Try to use as much mathematical language as possible (e.g. slope, equation, parallel, relation, etc.).

VERBAL PRESENTATION (5 MARKS)

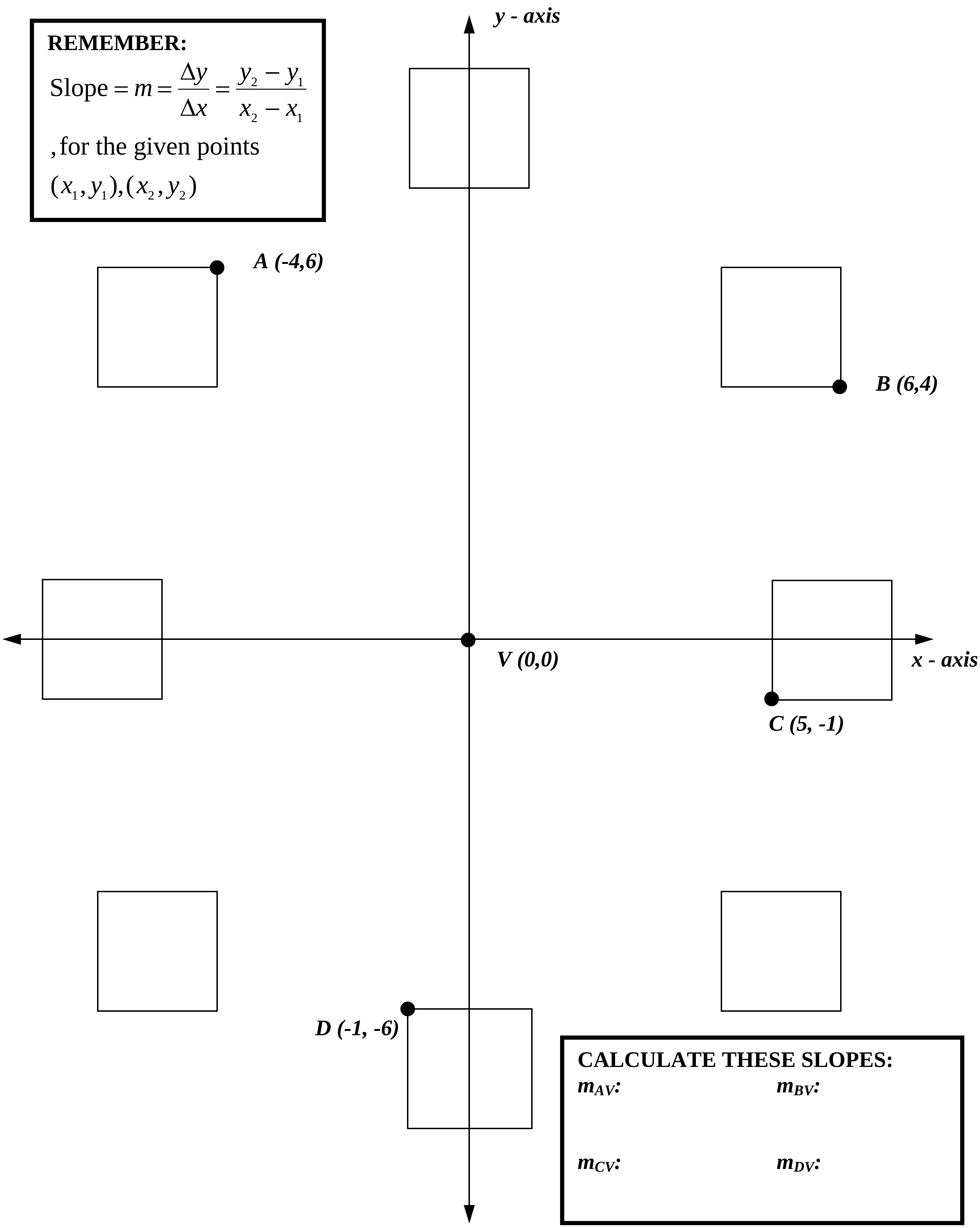
In a brief presentation to the class (3-5 minutes), simply display and describe your SMARTWORK. You may wish to discuss a few points from the planning stages (show preliminary sketches/ideas?) or from the process (changes made, new ideas, etc.). Relax. Enjoy the opportunity to share your creation with others.

ONE-POINT LINEAR PERSPECTIVE DRAWING WITH SLOPES

REMEMBER:

$$\text{Slope} = m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

, for the given points $(x_1, y_1), (x_2, y_2)$



CALCULATE THESE SLOPES:

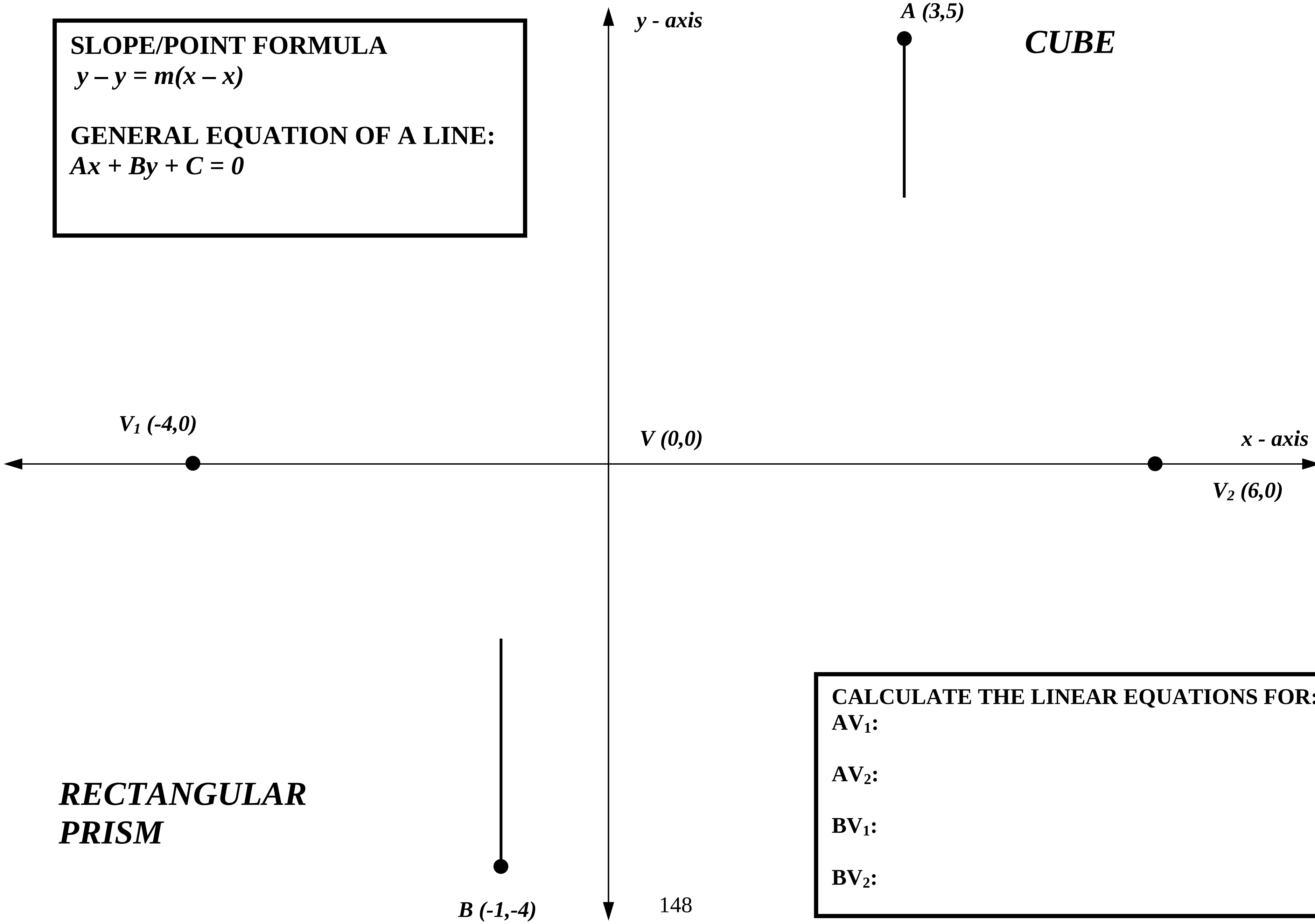
***m_{AV}*:** ***m_{BV}*:**

***m_{CV}*:** ***m_{DV}*:**

TWO-POINT LINEAR PERSPECTIVE DRAWING WITH LINEAR EQUATIONS

SLOPE/POINT FORMULA
y – y = m(x – x)

GENERAL EQUATION OF A LINE:
Ax + By + C = 0



CALCULATE THE LINEAR EQUATIONS FOR:

AV_1:

AV_2:

BV_1:

BV_2:

<u>Resources for Assignment # 2: *Linear Relations and Linear Perspective*</u>

Aston, M. (2000). *The panorama of the Renaissance.* New York, NY: Abradale Press.

Auvil, K. W. (1997). *Perspective drawing, 2nd Ed.* Mountain View, CA: Mayfield.

Corrain, L. (1997). *Masters of art: The art of the Renaissance.* Chicago, IL: Peter Bedrick Books.

Harper, R. (Producer/Director). (1991). *Masters of illusion.* [Video-tape].

Kemp, M. (1990). *The science of art: Optical themes in western art from Brunelleschi to Seurat.* London, UK: Yale University Press.

Leonardo da Vinci [Computer software]. (1998). Toronto, ON: Corbis Corporation.

Monk, C. H. (1975). *Leonardo da Vinci.* New York, NY: Hamlyn.

Richter, I. A. (1952). *The notebooks of Leonardo da Vinci.* New York, NY: Oxford University Press.

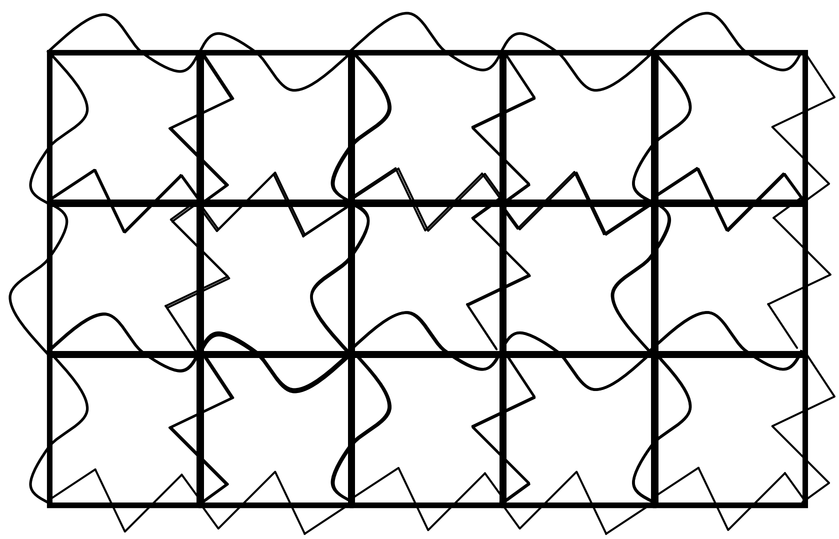
Romei, F. (1994). *Masters of art: Leonardo da Vinci.* Chicago, IL: Peter Bedrick Books.

Smith, R. (1995). *An introduction to perspective.* London, UK: Dorling Kindersley.

Thompson, M. (Producer/Director). (1997). *Leonardo da Vinci: Renaissance master.* [Video-tape].

Appendix G: Assignment 3—Geometric Patterning and Escher Tessellations

GEOMETRIC PATTERNING



ESCHER TESSELLATIONS ASSIGNMENT # 3

COURSE: MPM 1DA 01/02

DATE: DECEMBER 2000

STUDENT: ___________________________

RATIONALE AND RESOURCES FOR ASSIGNMENT # 3: TESSELLATIONS

RATIONALE

Mathematical knowledge becomes meaningful and powerful in application. This curriculum embeds the learning of mathematics in the solving of problems based on real-life situations. Other disciplines are a ready source of effective contexts for the study of mathematics. Rich problem-solving situations can be drawn from closely related disciplines, such as computer science, physics, or technology, as well as from subjects historically thought of as distant from mathematics, such as geography or art. It is important that these links between disciplines be carefully explored, analyzed, and discussed to emphasize for students the pervasiveness of mathematical knowledge and mathematical thinking in all subject areas.

The importance of communication in mathematics is a highlight of the elementary school curriculum and continues to be a highlight in secondary school. In all strands and all courses, expectations are included that require of students the clear and concise communication of reasoning or of findings. It is the expectation that students, over the course of their high school experience, will learn to write about their use of mathematics, effectively incorporating mathematical forms such as calculations, equations, graphs, or tables. This curriculum assumes a classroom environment in which students are called upon to explain their reasoning in writing, or orally to the teacher, to the class, or to other students in a group.

Excerpts from, *The Ontario Curriculum: Grades 9 and 10 Mathematics*, 1999, p. 4

MEASUREMENT & GEOMETRY (p. 16)

OVERALL EXPECTATION

- By the end of this course, students will formulate conjectures and generalizations about geometric relationships involving two-dimensional figures, through investigations facilitated by dynamic geometry software.

SPECIFIC EXPECTATIONS

- By the end of this course, students will illustrate and explain the properties of the interior and the exterior angles of triangles and quadrilaterals, and of angles related to parallel lines.
- By the end of this course, students will pose questions about geometric relationships, test them, and communicate the findings, using appropriate language and mathematical forms.

SELECTED RESOURCES

Bennett, D. (Ed.). (1999). *Exploring geometry with the Geometer's Sketchpad*. Emeryville, CA: Key Curriculum.

Ernst, B. (1976). *The magic mirror of M. C. Escher*. Toronto, Canada: Random House.

Locher, J. L., & Thé, E. (2000). *The magic of M.C.Escher*. New York, NY: Harry N. Abrams, Inc.

Ontario Ministry of Education and Training. (1999). *The Ontario Curriculum Grades 9 and 10: Mathematics.* Toronto, ON: Queen's Printer for Ontario.

Ranucci, E. R. (1977). Creating Escher-type drawings. Palo Alto, CA: Creative Publications.

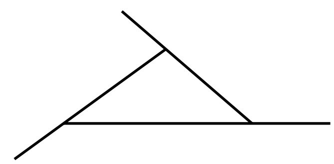
MATHEMATICS KNOWLEDGE & UNDERSTANDING

Define and/or illustrate the following terms: (1 MARK EACH)

10

REGULAR POLYGON (GIVE 3 EXAMPLES):

INTERIOR & EXTERIOR ANGLES (LABEL):



M. C. ESCHER:

TESSELLATION:

TRANSFORMATION:

TRANSLATION (SLIDE):

ROTATION (TURN):

REFLECTION (FLIP):

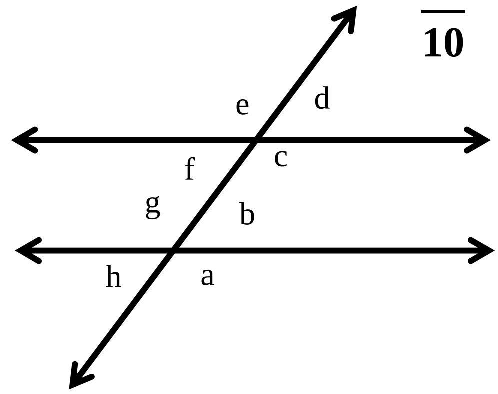
METAMORPHOSIS:

H. S. COXETER:

MATHEMATICS THINKING, INQUIRY, & PROBLEM SOLVING

Solve or complete the following questions:

10



BETWIXT & BETWEEN (3 MARKS)

Given: 2 parallel lines intersected by a transversal

Solve: If angle a is 123°, find the other seven angles:

a = 123° b = c = d =

e = f = g = h =

Alternate angles (e.g. b, f) and corresponding angles (e.g. b, d) are ______.

DETECTIVE POLYGON (4 MARKS)

REGULAR POLYGON	NUMBER OF SIDES/ANGLES	SUM OF INTERIOR ANGLES	MEASURE OF EACH INTERIOR ANGLE	WILL IT TESSELLATE?
triangle	3	180 °	60 °	YES
quadrilateral				
pentagon				
hexagon				
heptagon				
octagon				
n-gon	n	$(n-2)\,180°$	$\frac{(n-2)\,180}{n}°$	

To tessellate a plane, the interior angle of a regular polygon must divide evenly into ____ degrees.

PATTERN PATROL (3 MARKS)

Identify 3 patterns from the world around you that demonstrate these transformations:

TRANSFORMATION	*LOCATION OF THE PATTERN*	*THUMBNAIL SKETCH OF PATTERN*
TRANSLATION		
ROTATION		
REFLECTION		

MATHEMATICS APPLICATION OF CONCEPTS

Create the following items:

15 (with a line above it)

TESSELLATION PATTERNS (8 MARKS)

- Worksheet One: Tessellation by Translation
- Worksheet Two: Tessellation by Rotation
- Worksheet Three: Tessellation using a Template
- Worksheet Four: Tessellation using *Geometer's Sketchpad*

GEOMETRIC "SMARTWORK" (SECONDARY MATH AND ART WORK) (7 MARKS)

- Based on the new learning from this assignment and using the handouts and grids provided, construct a geometric SMARTWORK that features a tessellation pattern.
- This project may be 2- or 3-dimensional (e.g. polygons or polyhedra). It may be as simple as a single translation on a square, or be much more complex, featuring a variety of transformations such as translations, rotations, reflections, glide-reflections, midpoint rotations, or combinations of the above. Colour, medium, & size of the SMARTWORK are also yours to decide upon.
- The finished tessellation must possess a consistent pattern that completely tessellates the plane (i.e., avoid gaps filled with decorations!), and will also be assessed for creativity & complexity.

MATHEMATICS COMMUNICATION

Use mathematical language to communicate in the following forms:

___
15

WRITTEN DESCRIPTION OF YOUR "SMARTWORK" (10 MARKS)

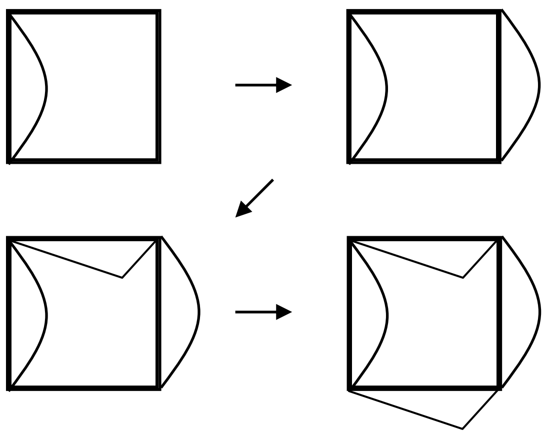
Use the remainder of this page (add further sheets if necessary) to describe in detail your finished project. For example, you should explain the regular polygon used as a basis for the pattern, as well as the type(s) of transformations that you used to construct your tessellation. Try to use as much mathematical language as possible (e.g. angles, translation, rotation, reflection, etc.).

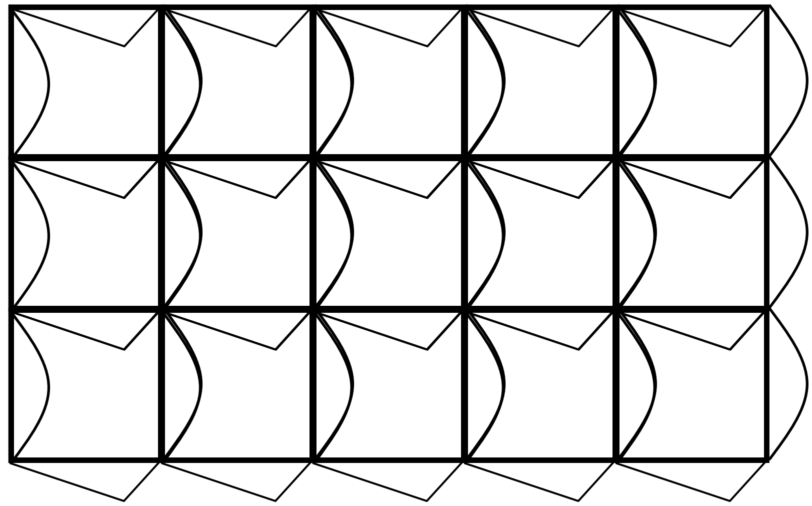
VERBAL PRESENTATION (5 MARKS)

In a brief presentation to the class (3-5 minutes), simply display and describe your SMARTWORK. You may wish to discuss a few points from the planning stages (show preliminary sketches/ideas?) or from the process (changes made, new ideas, etc.). Relax. Enjoy the opportunity to share your creation with others.

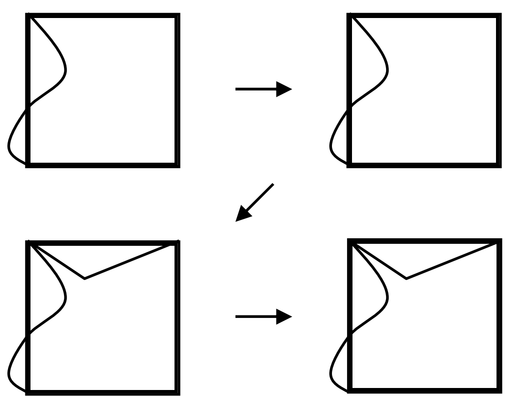
<u>**WORKSHEET 1: TESSELLATION BY TRANSLATION**</u>

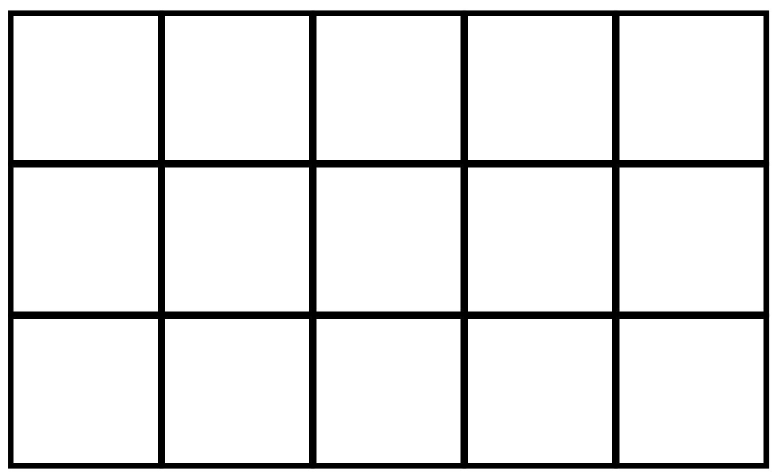
EXAMPLE



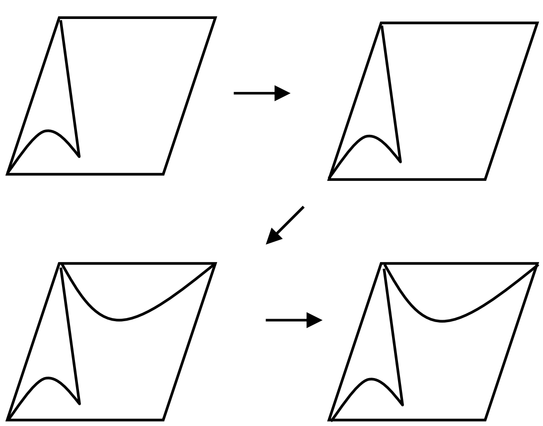


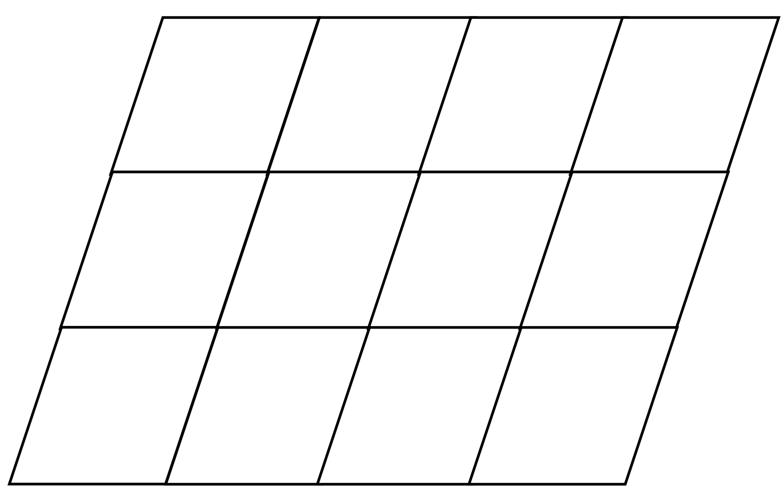
TRANSLATE & TESSELLATE





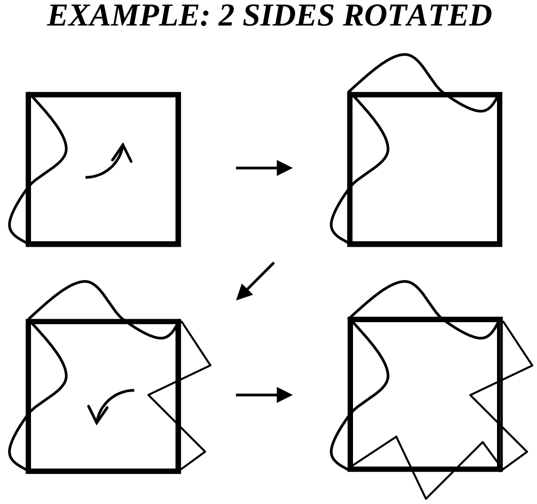
TRANSLATE & TESSELLATE

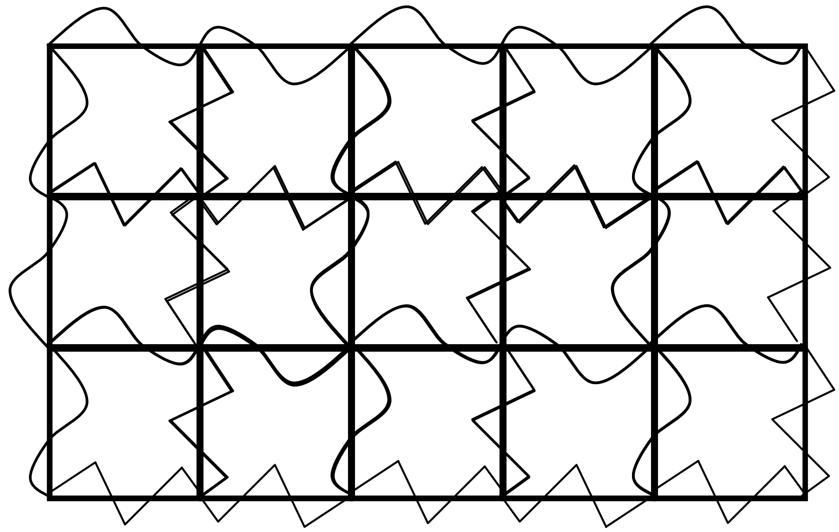




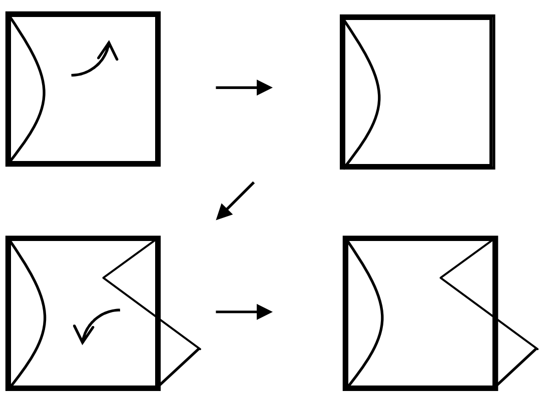
WORKSHEET TWO: TESSELLATION BY ROTATION

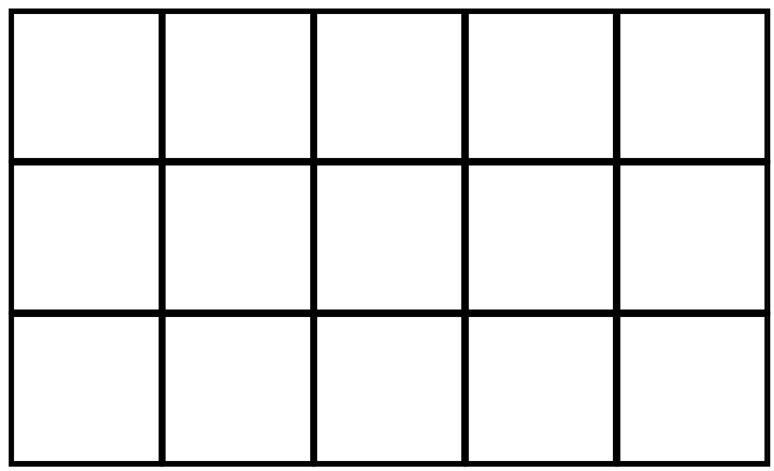
EXAMPLE: 2 SIDES ROTATED



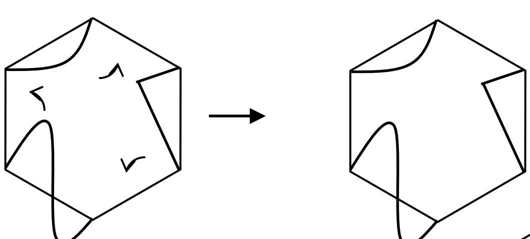


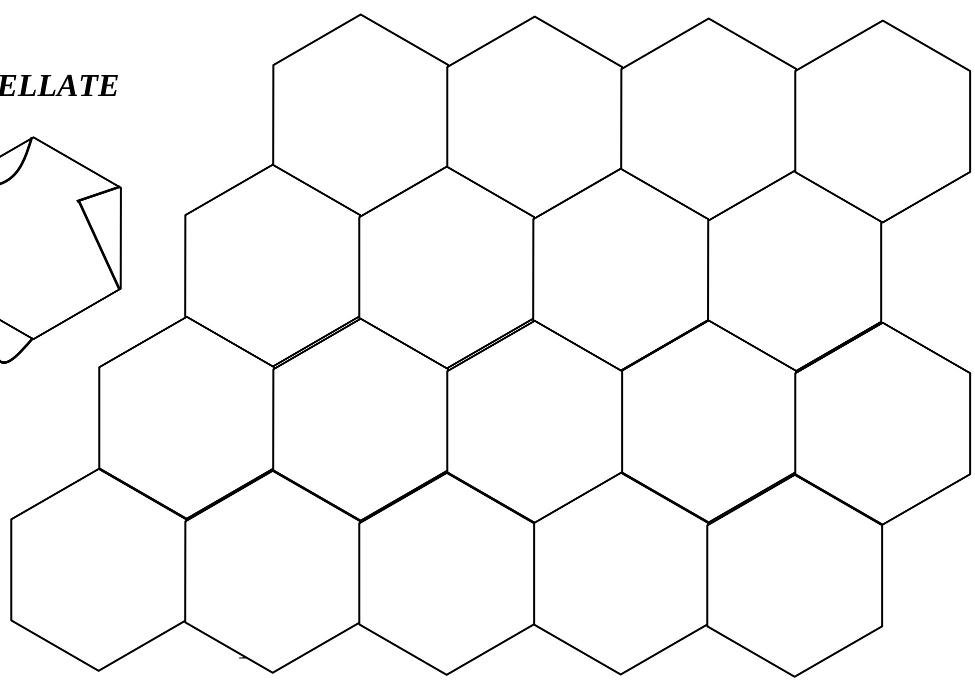
ROTATE 2 SIDES & TESSELLATE





ROTATE 3 SIDES & TESSELLATE





Midpoint rotations & glide reflections also lead to many interesting designs. Experiment using a variety of grids.

WORKSHEET THREE: TESSELLATION USING A TEMPLATE

Instructions: First, create an original template using the scissors, tape, and thick paper provided. Then, trace this geometric shape to develop a tessellating pattern. Depending on your original transformations during construction, the template may need to be rotated, translated, reflected, or a combination of the above. The template itself will guide you in the proper placement of each tessellating piece on the plane. Apply two different colours to adjacent shapes for visual clarity.

WORKSHEET FOUR: TESSELLATION USING "SKETCHPAD"

Instructions:

- Using *Geometer's Sketchpad* and the algorithms provided, create a dynamic tessellation pattern.
- Save a copy of this creation, using your name, to the designated drive and file folder.
- Print out a hard copy and paste the pattern in the rectangle below.

<u>Resources for Assignment # 3: *Geometric Patterning and Escher Tessellations*</u>

Acorn Media Publishing. (1999). *The life and works of M. C. Escher*. [Video-tape].

Atlas Video, Inc. (1994). *The fantastic world of M. C. Escher.* [Video-tape].

Bennett, D. (Ed.). (1999). *Exploring geometry with the Geometer's Sketchpad.* Emeryville, CA: Key Curriculum Press.

Bezuszka, S., Kenney, M., & Silvey, L. (1977). *Tessellations: The geometry of patterns.* Palo Alto, CA.: Creative Publications.

Ernst, B. (1976). *The magic mirror of M. C. Escher.* Toronto, Canada: Random House.

Escher, M. C. (1967). *The graphic work of M. C. Escher.* New York, NY: Ballantine Books.

Escher, M. C. (1971). *The world of M. C. Escher.* New York, NY: H. N. Abrams.

Escher, M. C. (1989). *Escher on Escher: Exploring the infinite.* New York, NY: Abrams.

Hofstadter, D. R. (1979). *Godel, Escher, Bach: An eternal golden braid.* New York, NY: Basic Books Inc.

Locher, J. L., & Thé, E. (2000). *The magic of M.C.Escher.* New York, NY: Harry N. Abrams.

Ranucci, E. R. (1977). *Creating Escher-type drawings.* Palo Alto, CA: Creative Publications.

Schattschneider, D. (1990). *M. C. Escher: Visions of symmetry.* New York, NY: Freeman & Co.

Seymour, D., & Britton, J. (1989). *Introduction to tessellations.* Palo Alto, CA: Dale Seymour Publications.

Shaffer, D. W. (1997). Learning mathematics through design: The anatomy of Escher's world. *Journal of Mathematical Behaviour, 1*<u>6</u>(2), 95-112.

Appendix H: Final Assessment Rubric

GRADE 9 ASSESSMENT RUBRIC FOR THREE INTEGRATED ASSIGNMENTS *STUDENT:*

Expectations	*Criteria*	Level 1	Level 2	Level 3	Level 4
	Knowledge/Understanding				
	The student:				
4, 9	defines relevant terms and names accurately	defines relevant terms and names with limited accuracy	defines relevant terms and names with some accuracy	defines relevant terms and names with considerable accuracy	defines relevant terms and names with a high degree of accuracy
	Thinking/ Inquiry/ Problem Solving				
	The student:				
1, 2, 3, 9	solves multi-step problems involving applications of ratio, slope, and geometry with accuracy	solves multi-step problems involving applications of ratio, slope, and geometry with limited accuracy	solves multi-step problems involving applications of ratio, slope, and geometry with some accuracy	solves multi-step problems involving applications of ratio, slope, and geometry with considerable accuracy	solves multi-step problems involving applications of ratio, slope, and geometry with a high degree of accuracy
4	researches examples of the learning from everyday life with completeness	researches examples of the learning from everyday life with limited completeness	researches examples of the learning from everyday life with some completeness	researches examples of the learning from everyday life with considerable completeness	researches examples of the learning from everyday life with a high degree of completeness
	Communication				
	The student:				
4	communicates projects clearly in writing, using mathematical language, symbols, and units	communicates projects in writing, using mathematical language, symbols, and units with limited clarity	communicates projects in writing, using mathematical language, symbols, and units with some clarity	communicates projects in writing, using mathematical language, symbols, and units with considerable clarity	communicates projects in writing, using mathematical language, symbols, and units with a high degree of clarity
4	communicates projects clearly to the class using mathematical language	communicates projects to the class using mathematical language with limited clarity	communicates projects to the class using mathematical language with some clarity	communicates projects to the class using mathematical language with considerable clarity	communicates projects to the class using mathematical language with a high degree of clarity
	Application				
	The student:				
1, 5, 6, 7, 8	applies concepts and procedures accurately by hand and using technology	applies concepts and procedures by hand and using technology with limited accuracy	applies concepts and procedures by hand and using technology with some accuracy	applies concepts and procedures by hand and using technology with considerable accuracy	applies concepts and procedures by hand and using technology with a high degree of clarity
4, 10	creates SMARTWORK*S* with originality, complexity, and completeness	creates SMARTWORK*S* with limited originality, complexity, and completeness	creates SMARTWORK*S* with some originality, complexity, and completeness	creates SMARTWORK*S* with considerable originality, complexity, and completeness	creates SMARTWORK*S* with a high degree of originality, complexity, and completeness

**** The expectations from the Ontario Curriculum (1999) that correspond to the numbers in this chart are listed, by mathematical strand, on the reverse side of this sheet.***

EXPECTATIONS ADDRESSED IN THE INTEGRATED ASSIGNMENTS

These three assignments gave students the opportunity to demonstrate achievement of the following selected expectations from three mathematical strands – *Number Sense and Algebra; Analytic Geometry;* and *Measurement and Geometry.*

NUMBER SENSE AND ALGEBRA

Students will:

1 - solve multi-step problems requiring numerical answers, using a variety of strategies and tools. (NAV.01)

2 - solve multi-step problems involving applications of percent, ratio, and rate as they arise throughout the course. (NA1.03)

3 - use a scientific calculator effectively for applications that arise throughout the course. (NA1.04)

4 - communicate solutions to problems in appropriate mathematical forms and justify the reasoning used in solving the problems. (NA4.03)

ANALYTIC GEOMETRY

Students will:

5 - determine the slope of a line segment, using various formulas. (AG2.01)

6 - plot points on the xy-plane and use the terminology and notation of the xy-plane correctly. (AG3.01)

7 - determine the equation of a line, given information about the line. (AG3.04)

MEASUREMENT AND GEOMETRY

Students will:

8 - formulate conjectures and generalizations about geometric relationships involving two-dimensional figures, through investigations facilitated by dynamic geometry software, where appropriate. (MGV.03)

9 - illustrate and explain the properties of the interior and the exterior angles of triangles and quadrilaterals, and of angles related to parallel lines. (MG3.01)

10 - pose questions about geometric relationships, test them, and communicate the findings, using appropriate language and mathematical forms. (MG3.04)

Appendix I: Student Questionnaire

Appendix I

QUESTIONNAIRE REGARDING INTEGRATED ASSIGNMENT

NAME: ________________ **ASSIGNMENT:__________________**

1. How did you feel about the assignment when it was first presented to you in class?

2. Now that you have completed the assignment what are your feelings about it?

3. What did you learn from the assignment about mathematics and/or visual arts?

4. Did you feel that the assignment helped you to *creatively apply* your mathematics learning in the classroom?

5. Did you feel that the assignment helped you to *communicate* the mathematics that you have learned?

6. In your opinion, did the integration of the two topics help reinforce your math learning or distract from it?

7. Tell me about any ideas or solutions you may have had with this integrated assignment.

8. Tell me about any problems or frustrations you may have had with the assignment.

9. Do you feel that a bigger variety of examples should be shown before the commencement of the assignment, or would this hamper creativity?

10. What recommendations do you have for improving this project, or making it a better learning experience for you?

11. Tell me what you did not like about the assignment.

12. Tell me what you liked best about the assignment.

[Questionnaire modified from that of researcher Susan Schramm in her 1997 paper entitled *Related Webs of Meaning between the Disciplines: Perceptions of Secondary Students Who Experienced an Integrated Curriculum*]

N.B. The actual questionnaire was four pages in length, with spacing in between each of the twelve questions, allowing for adequate room for student comments.

Appendix J: Final Student Survey/Questionnaire

Appendix J

FINAL SURVEY/QUESTIONNAIRE REGARDING THE ASSIGNMENTS

STUDENT NAME: **DATE:**

Introduction
You have now experienced three integrated Mathematics & Visual Arts assignments. Please take a moment to visually locate your own SMARTWORKS around the classroom and to reflect on your efforts made in producing them. Also, look through your written assignments that I have handed back. Now, you are ready to complete this final reflective survey/questionnaire regarding these experiences.

PART ONE: SURVEY
1. Prior to attending this secondary school, one or more of my elementary school (K-8) teachers had covered the following topics in class: [check only the boxes that apply to you]

☐ Golden Ratio Constructions ☐ Linear Perspective Drawings ☐ Tessellation Drawings

2. In the following questions, check the one box that indicates your choice of assignment.

Question	Assign 1: Gold Sect	Assign 2: Lin Perspec	Assign 3: Tessellations
(a) My most favourite of the 3 assignments:	☐	☐	☐
(b) My least favourite of the 3 assignments:	☐	☐	☐
(c) Required the most amount of effort:	☐	☐	☐
(d) Most interesting to see others' works:	☐	☐	☐
(e) Best reinforced the new math learning:	☐	☐	☐
(f) I was most creative in this assignment:	☐	☐	☐
(g) I best communicated in this assignment:	☐	☐	☐
(h) I was most thorough in this assignment:	☐	☐	☐
(i) The teacher presented this assignment to me in the most effective & interesting way:	☐	☐	☐

3. Each of the three assignments featured the seven different types of activities listed below. Please rank the activities from most favourite (1) to least favourite (7) in the spaces provided.

___ Definitions
___ Problem Solving (Thinking & Inquiry)
___ Home Research & Calculations
___ Application Practice (Worksheets)
___ SMARTWORKS
___ Written Communication
___ Verbal Communication

For questions # 4-14, please choose your answer by making a check in the appropriate box.

4. The three integrated assignments helped me to *creatively apply* the new math learning in each.

☐ Strongly Agree ☐ Agree ☐ Disagree ☐ Strongly Disagree

5. The three integrated assignments helped me to better *communicate* (written & verbally) the new math learning in each.

☐ Strongly Agree ☐ Agree ☐ Disagree ☐ Strongly Disagree

6. The three integrated assignments increased my *motivation for learning* in this course.

☐ Strongly Agree ☐ Agree ☐ Disagree ☐ Strongly Disagree

7. The *groupwork* sessions were very helpful in completing the three integrated assignments.

☐ Strongly Agree ☐ Agree ☐ Disagree ☐ Strongly Disagree

8. The *exhibition of student work* in the classroom throughout the course was an important part of the three integrated assignments, regarding the motivation of students.

☐ Strongly Agree ☐ Agree ☐ Disagree ☐ Strongly Disagree

9. *Internet research* should be a required component of the three integrated assignments.

☐ Strongly Agree ☐ Agree ☐ Disagree ☐ Strongly Disagree

10. *Geometer's Sketchpad* activities should be required components of the three integrated assignments.

☐ Strongly Agree ☐ Agree ☐ Disagree ☐ Strongly Disagree

11. The three integrated assignments were a negative *distraction* from the math learning in this course.

☐ Strongly Agree ☐ Agree ☐ Disagree ☐ Strongly Disagree

12. If given the choice, I would have preferred three assignments that had no *visual arts* and *historical* components (i.e. no SMARTWORKS, historical information, videos, handouts, classroom displays, etc.) and that simply featured various types of questions from the different topics being studied.

☐ Strongly Agree ☐ Agree ☐ Disagree ☐ Strongly Disagree

13. This integrated approach to math education, using Visual Arts, should be used regularly.

☐ Strongly Agree ☐ Agree ☐ Disagree ☐ Strongly Disagree

14. This integrated approach to math education, using Visual Arts, should not be used.

☐ Strongly Agree ☐ Agree ☐ Disagree ☐ Strongly Disagree

PART TWO: QUESTIONNAIRE

1. What did you not like about the three integrated assignments?

2. What did you like most about the three integrated assignments?

3. I offer the following suggestion(s) to make these learning experiences even more effective… (Please use the back of this sheet if more space is required.)

Appendix K: Semi-Structured Interview Questions for Grade 11 Students

Appendix K

SEMI-STRUCTURED INTERVIEW FORMAT: GR. 11 PILOT STUDY

After briefing the student on the present Gr. 9 research and reminding him/her about the pilot study of which he/she was a part, ask and record the following questions:

[I] In your opinion, did the *creative application* of the new learning help you better understand the math concepts of ratio & proportion?

[II] In your opinion, did the *communication (written & oral)* of the projects help you better understand the concepts of ratio & proportion?

[III] In your opinion, how did the integrated assignment affect your *motivation* regarding the course?

[IV] In your opinion, how important was the *display of student work* in the classroom regarding student motivation?

[V] In your opinion, was the Griffith's article entitled, *Mathematics at the Turn of the Century*, a good springboard for this type of open-ended assignment?

[VI] In your opinion, did the integration of mathematics and visual arts increase your overall *understanding* of math concepts or did it distract from it?

[VII] What did you *like* about the integrated assignment?

[VIII] What did you *dislike* about the integrated assignment?

Appendix L: Semi-Structured Interview Questions for Grade 9 Students

Appendix L

SEMI-STRUCTURED INTERVIEW FORMAT: GR. 9 STUDENTS

After drawing the student's attention to their three submitted assignments and to their displayed SMARTWORKS, the following questions would begin:

[I] In this course you have completed three integrated assignments: [i] Golden Section - Proportion; [ii] Renaissance Perspective Drawing - Linear Relations; & [iii] Escher Tessellations - Geometric Patterning. Tell me how you feel, in general, regarding these three integrated assignments?

[II] In your opinion, did the *creative application* of the new learning help you to better understand the math concepts of proportion, line, and pattern?

[III] In your opinion, did the *communication (written & oral)* of the SMARTWORKS help you better understand the math concepts of proportion, line, & pattern?

[IV] In your opinion, how did the integrated assignments affect your *motivation* in this course?

[V] In your opinion, how important was the *display of student work* in the classroom regarding student motivation?

[VI] In your opinion, did the integration of mathematics and visual arts increase your overall *understanding* of math concepts or did it distract from it?

[VII] In your opinion, were the groupwork sessions a valuable part of the integrated assignments?

[VIII] In your opinion, was the Open House an important part of the integrated assignments?

[IX] What did you *like* about the three integrated assignments?

[X] What did you *dislike* about the three integrated assignments?

[N.B. Additional probing to prompt and clarify questions may take place during the interviews.]

Appendix M: Letter/Response Form to Parent/Guardian Regarding Open House

Appendix M

PARENT/GUARDIAN INVITATION AND RESPONSE FORM

Mr. Daniel Jarvis
Math Teacher, ________

January 2001

Re: Integrated Assignments and Open House

Dear Parent/ Guardian,

As this semester draws to a close, I would like to thank you for your ongoing support in regards to your child's mathematical learning. It has been a very full and challenging new curriculum, and the students are now preparing for their final exam.

As you know, the students in Grade 9 Academic mathematics have completed three integrated assignments as part of that curriculum. The resulting projects (SMARTWORKS) are now on display in our math room at ______. As you may be interested in seeing for yourself the final products, or in discussing the integrated learning with me in person, you are cordially invited to drop in during an *Open House* on Tuesday January 23rd, from 6:00 to 9:00 p.m. Refreshments will be provided and your child is also welcomed (encouraged) to attend and act as your private tour-guide.

Finally, I am very much interested in how these assignments were perceived by you, the parent/guardian. Your comments and/or constructive criticism would be much appreciated, as it would help me to see the research in yet another important light. Like the student questionnaires, your comments will be confidential and will in no way affect your child's achievement in this course. Please feel free to contact me personally by phone or email, or clip and return the following form. Thank you.

Yours in Education,
Mr. Daniel Jarvis

Parent/Guardian Response Form

Comments Regarding Three Assignments (Please use further sheets if necessary):

____
____
____
____
____
____

Signature: ____________________________

Appendix N: An Integrated History of the Calculus Assignment—Fluxions and Deductions

FLUXIONS
&
DEDUCTIONS

AN INTEGRATED HISTORY OF THE CALCULUS ASSIGNMENT

COURSE: MCA OAA 01

DATE: MARCH 2001

STUDENT: ___________________________

RATIONALE/RESOURCES FOR: FLUXIONS & DEDUCTIONS

RATIONALE

Mathematics provides a means of communication that is powerful, concise, and usually unambiguous. Through mathematics it is possible to represent complex situations, explain events, and predict outcomes. Mathematics is being used increasingly to create models for analysis and problem solving in a variety of fields, such as science, economics, medicine, art, music, and many of the social sciences.

Mathematics provides a way of thinking that involves the study of patterns, the creation of abstract systems, and the use of logical arguments. The appreciation of the order and structure of a mathematical proof or a mathematical system can be compared to enjoyment of the arts. Just as works of art are interpreted in a variety of ways by different individuals, so the appreciation of much of mathematics is, ultimately, a matter of personal interpretation.

Excerpts from, *Curriculum Guideline: Mathematics, Intermediate & Senior Divisions*, 1985, p. 6

Mathematical knowledge becomes meaningful and powerful in application. This curriculum embeds the learning of mathematics in the solving of problems based on real-life situations. Other disciplines are a ready source of effective contexts for the study of mathematics. Rich problem-solving situations can be drawn from closely related disciplines, such as computer science, physics, or technology, as well as from subjects historically thought of as distant from mathematics, such as geography or art. It is important that these links between disciplines be carefully explored, analyzed, and discussed to emphasize for students the pervasiveness of mathematical knowledge and mathematical thinking in all subject areas.

The importance of communication in mathematics is a highlight of the elementary school curriculum and continues to be a highlight in secondary school. In all strands and all courses, expectations are included that require of students the clear and concise communication of reasoning or of findings. It is the expectation that students, over the course of their high school experience, will learn to write about their use of mathematics, effectively incorporating mathematical forms such as calculations, equations, graphs, or tables. This curriculum assumes a classroom environment in which students are called upon to explain their reasoning in writing, or orally to the teacher, to the class, or to other students in a group.

Excerpts from, *The Ontario Curriculum: Grades 11 & 12 Mathematics*, 2000, pp. 3-4

MCA OAA Curriculum Guideline Expectations (1985)

APPLICATIONS OF DERIVATIVES: CURVE SKETCHING (p. 74)
4. (h) Approximating solutions to equations f(x) = 0 by Newton's Method

ANTIDIFFERENTIATION: APPLICATIONS OF DIFFERENTIAL EQUATIONS (p. 74)
2. (a) Solving problems such as those involving cost, revenue, mixing, rates of cooling, Newton's law of cooling, and rates of growth

SELECTED RESOURCES

Berlinski, D. (1995). *A tour of the Calculus*. New York, NY: Pantheon Books.

Boyer, C. B. (1991). *A history of mathematics, 2nd Ed.* Toronto, ON: John Wiley & Sons.

Ontario Ministry of Education. (1985). *The curriculum guideline: Mathematics, Intermediate and Senior divisions.* Toronto, ON: Author.

Ontario Ministry of Education and Training. (2000). *The Ontario curriculum Grades 11 & 12: Mathematics*. Toronto, ON: Queen's Printer for Ontario.

MATHEMATICS KNOWLEDGE & UNDERSTANDING

1. Identify the following characters and briefly describe their contribution(s) to the development of the Calculus: (1 MARK EACH)

10

ARCHIMEDES:

FERMAT:

BARROW:

NEWTON:

LEIBNIZ:

BERNOULLI (JACOB & JOHANN):

RIEMANN:

CAUCHY:

2. Briefly describe the two main branches of modern Calculus: (1 MARK EACH)

DIFFERENTIAL CALCULUS:

INTEGRAL CALCULUS:

MATHEMATICS THINKING, INQUIRY, & PROBLEM SOLVING

Solve or complete the following questions:

10

NEWTON'S LEGACY (3 MARKS)

In the winter of 1665-66, Trinity College in Cambridge closed its doors owing to the darkness of the Great, or bubonic, plague. Newton returned to his home in the English countryside. In the year that followed, Newton, at age twenty-three, made four major discoveries that would shape his world for the next three hundred years. In English history, those twelve months were commonly known as the *annus mirabilis*, the year of miracles.

List the four major discoveries made by the brilliant young Newton:

[I] [II]

[III] [IV]

Name Newton's most famous work, published in 1687, which presented both his astronomical and dynamical discoveries in one complex theory of the universe.

NEWTON'S METHOD (3 MARKS)

This method is used to approximate roots of equations for situations where standard techniques are either too complicated or non-existent. It is called an iterative method, wherein one successively calculates the terms of a sequence of approximations to the solution. The equation for x_{n+1}, which relates successive terms in the sequence, is called a recursion formula, and the one for Newton's Method is the following:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, n = 1,2,\ldots, \text{ with } x_1 \text{ suitably chosen.}$$

Use Newton's Method to approximate a root of $f(x) = x^3 - 2x - 5 = 0$, starting with $x_1 = 2$ (to 5 decimal places).

NEWTON'S LAW OF COOLING (4 MARKS)

Newton's Law of Cooling states that the rate at which a body cools to the temperature of its surroundings is proportional to the temperature difference between the body and its surroundings. The formula used is $T = A + (T_o - A)\, e^{kt}$ where A is the temperature of the surroundings, T_o is the initial temperature of the body, and k is a characteristic of the material of the body. Solve the following problem using Newton's Law of Cooling:

The temperature of a cup of coffee is 80°C when brought into a room where the temperature is 22°C. After 10 minutes, the temperature of the coffee is 60°C. How long will it take for the temperature to reach 30°C?

MATHEMATICS APPLICATION OF CONCEPTS

10

Creatively apply the new learning:

"I do not know what I may appear to be to the world, but to myself I seem to have been only like a boy playing on the sea-shore, and diverting myself in now and then finding a smoother pebble or a prettier shell than ordinary, whilst the great ocean of truth lay all undiscovered before me."

Sir Isaac Newton (1642-1727)

"I would walk twenty miles to my worst enemy if I could learn something from it."

Gottfried Wilhelm Leibniz (1646-1716)

A BAROQUE "SMARTWORK" (SECONDARY MATH AND ART WORK) (10 MARKS)

The exciting historical period from 1600-1750, often referred to in music and visual arts as the Baroque, featured monumental achievements and advances in all the disciplines. It was the time of Bach, Vivaldi, Handel and Scarlatti who revolutionized the musical world. In literature, authors such as Shakespeare, Marlowe, Milton and Jonson were redefining poetry and prose. In the world of art, Caravaggio, Zurbaran, Poussin, Rubens, Vermeer and Rembrandt were painting in a powerful new genre involving a sense of movement, light/dark contrasts, infinite space, high realism and a deep sense of spirituality. The Roman Catholic church was a highly influential patron of the arts, as was its Counter Reformation, employing the arts as a means of spreading the faith. And finally, scientific and mathematical minds such as Galileo, Barrow, Newton, & Leibniz were issuing in a modern world with proverbial leaps and bounds.

Based on the new learning from this assignment, particularly with respect to the contributions made by Sir Isaac Newton, design a Baroque SMARTWORK that juxtaposes a selection of ideas form this period that interest you the most. This project may take any form, using any media, and is thereby wide open to personal interpretation. Some ideas might include a poster, a sculpture, a collage, a mobile, a mathematical or physical model or set of questions with diagrams, a song, a dramatic narrative, a costume, etc.

Your imagination is the limit. Push this limit. Develop something uniquely original to you, as you absorb all the richness and explosive energy of the Baroque era. There should be an obvious tie to the Calculus somewhere within the overall design of your SMARTWORK.

Good luck and go for Baroque.

MATHEMATICS COMMUNICATION

Use mathematical language to communicate in the following forms:

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10

WRITTEN DESCRIPTION OF YOUR "SMARTWORK" (5 MARKS)

Use the remainder of this page (add further sheets if necessary) to describe in detail your finished project. For example, you should try to use as much mathematical language as possible (e.g. differentiation, laws, methods, integration, fluxions, etc.) wherever appropriate in explaining your final design.

VERBAL PRESENTATION (5 MARKS)

In a brief presentation to the class (2-3 minutes), simply display and describe your SMARTWORK. You may wish to discuss a few points from the planning stages (show preliminary sketches/ideas?) or from the process (e.g. changes made, new ideas, etc.). Relax. Enjoy the opportunity to share your creation with others.

Appendix O: Learning Between the Lines – Digital Slideshow [CD-ROM]

Appendix P: Learning Between the Lines – Digital Video Documentary [CD-ROM]